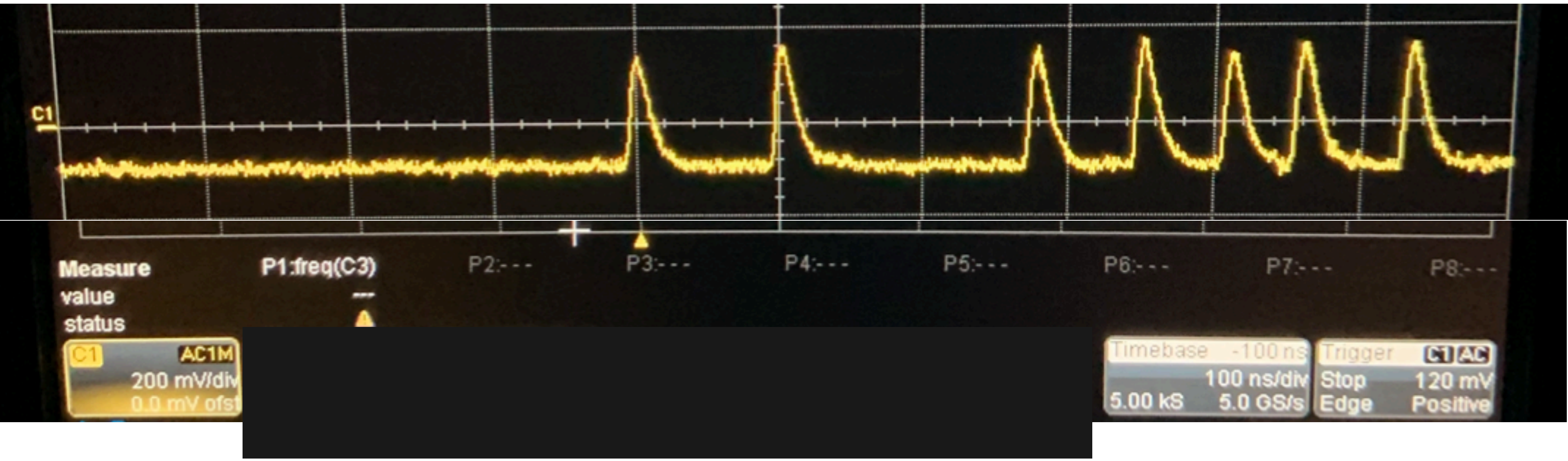
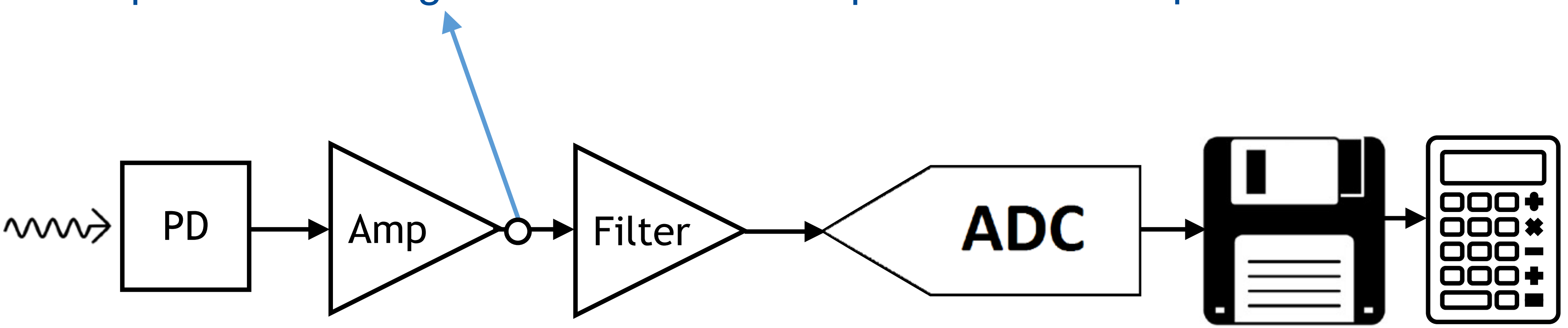
PHYS127AL Lecture 2

David Stuart, UC Santa Barbara

Input and output impedance; AC signals; capacitors



Example of an AC signal from a burst of 7 photons over ~1 μs.



Review and outline

Review

Resistors and Ohm's law

Thevenin and Norton equivalent circuits

Outline

Impedance rule for circuit stages: high input, & low output, impedance

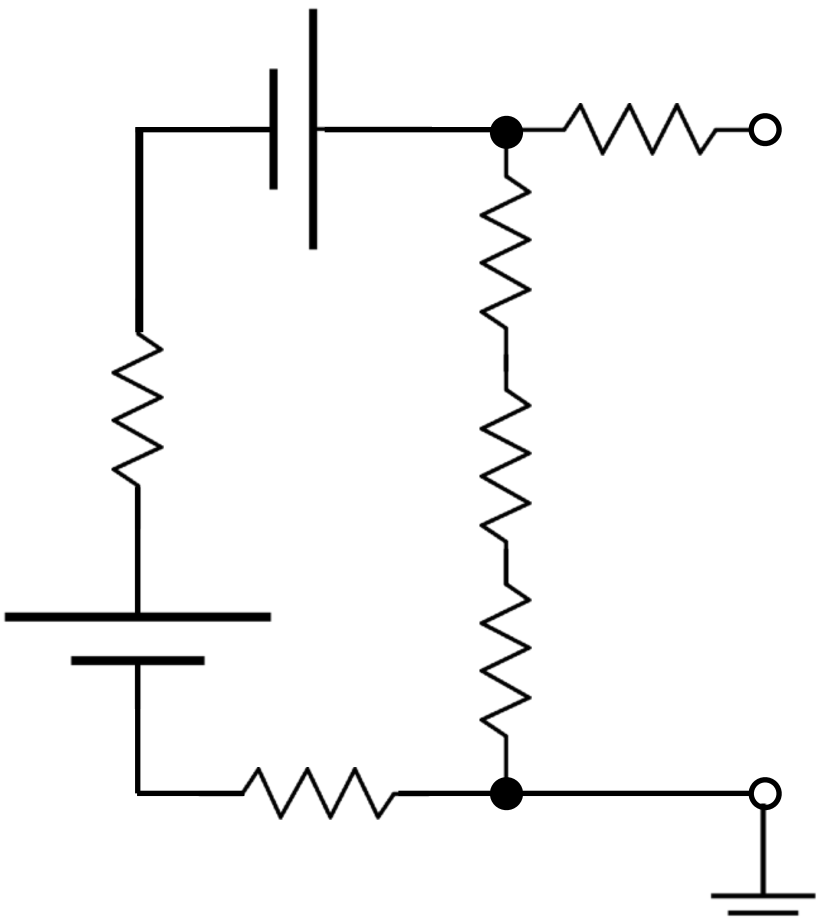
Calculating impedance of circuit stages

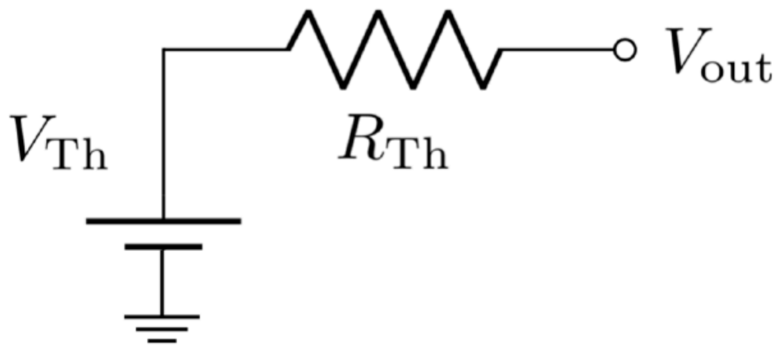
DC vs AC and time dependent signals

Capacitors

Thevenin Equivalent Circuits

Any mix of voltage sources and resistors can be treated as equivalent to a single voltage source V_{Th} and a single resistor R_{Th}.





Thevenin and Norton Equivalent Circuits

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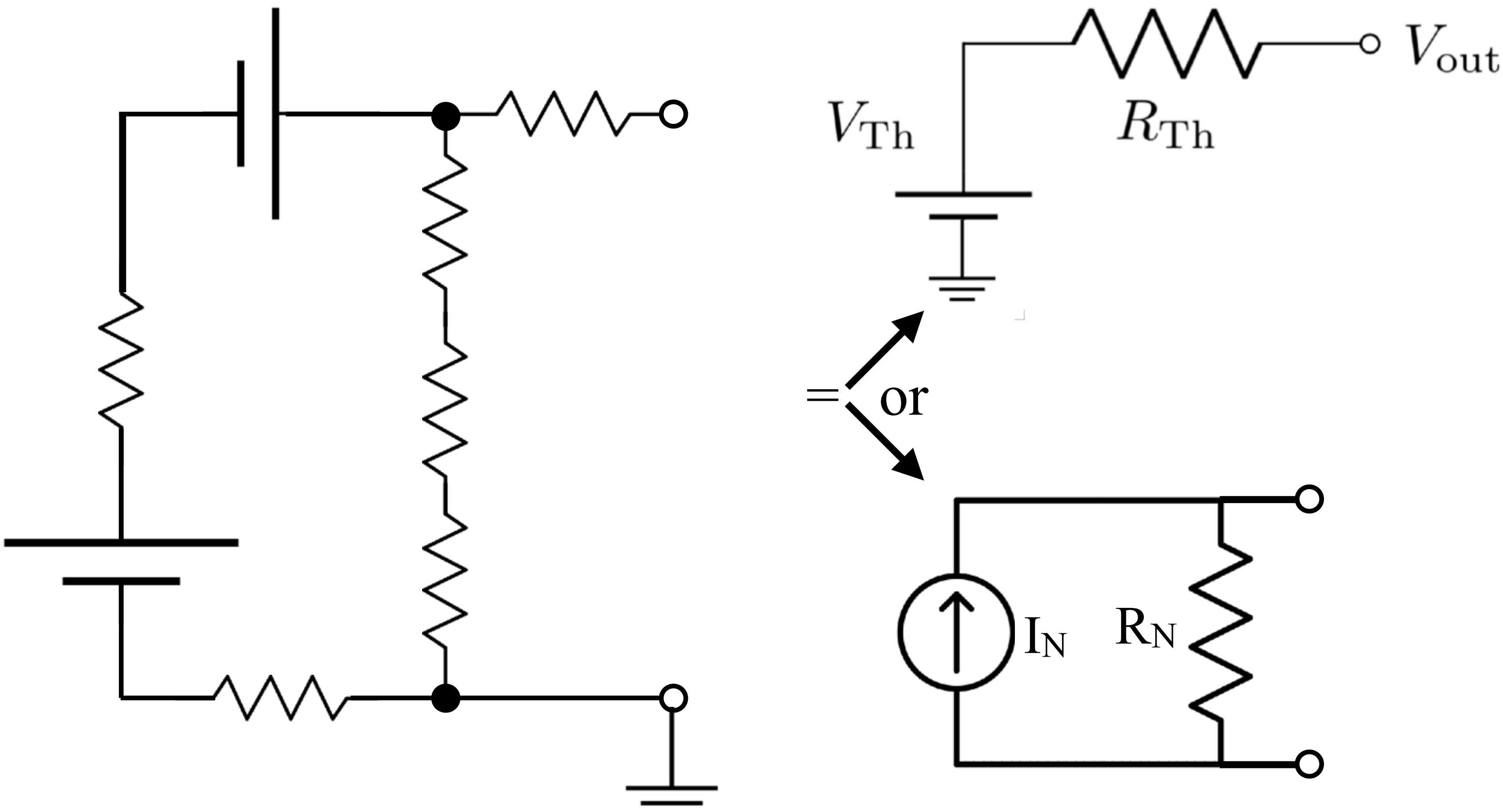

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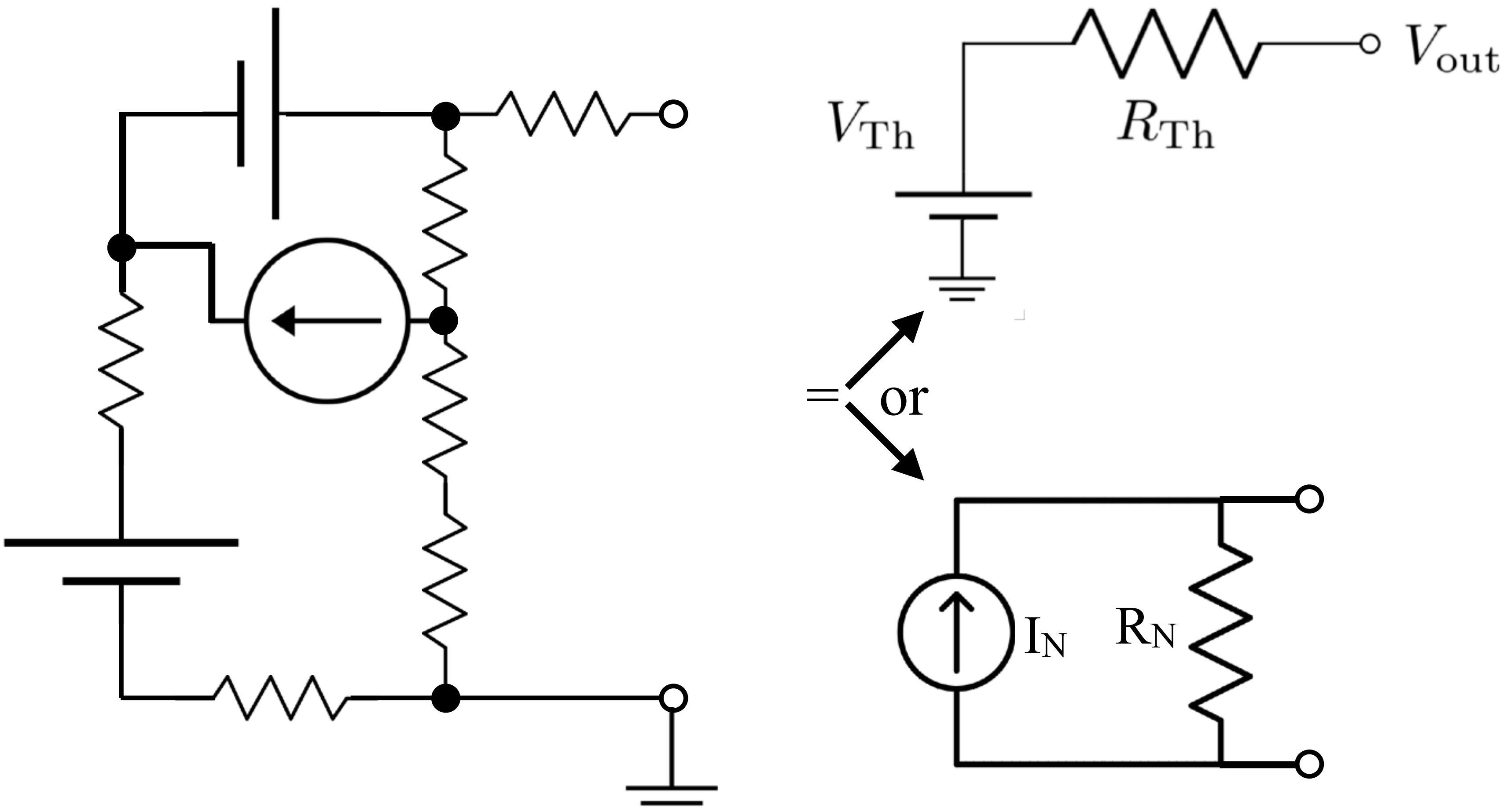
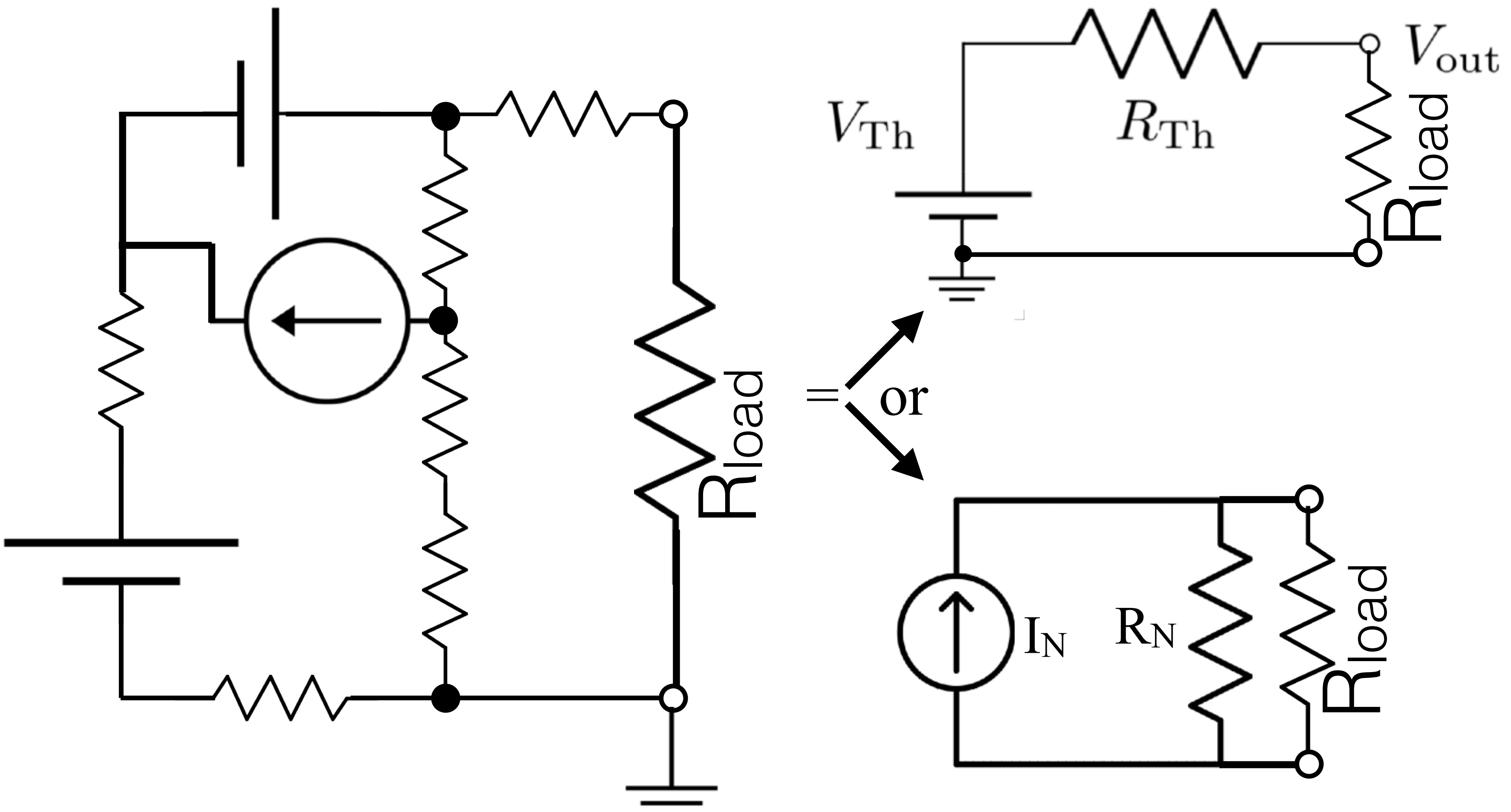


Thevenin and Norton Equivalent Circuits

This means that all three behave them same in terms of the IV characteristics when loaded by a resistor. $I(R_{Load})$ and $V(R_{Load})$ is the same for all three circuits.



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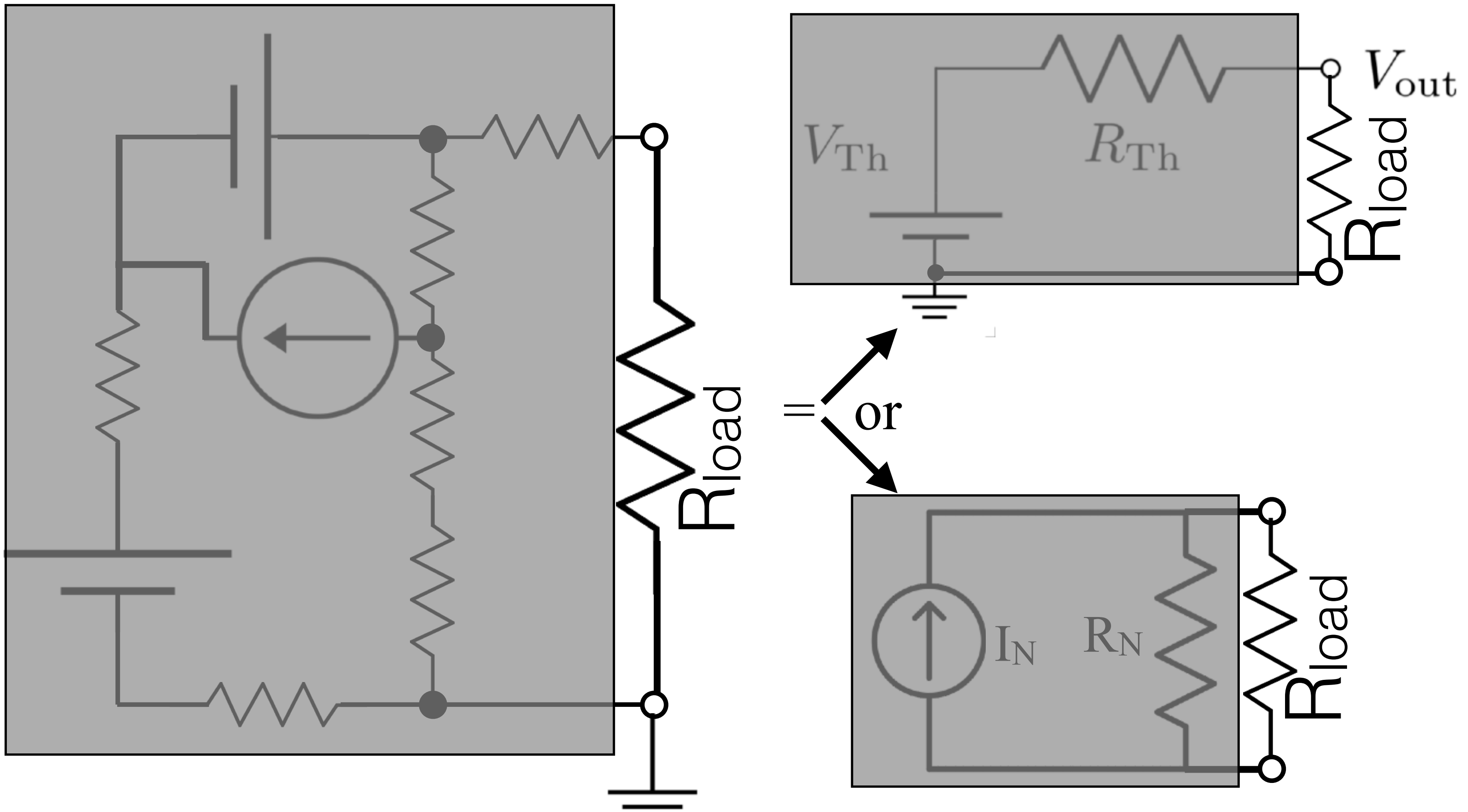

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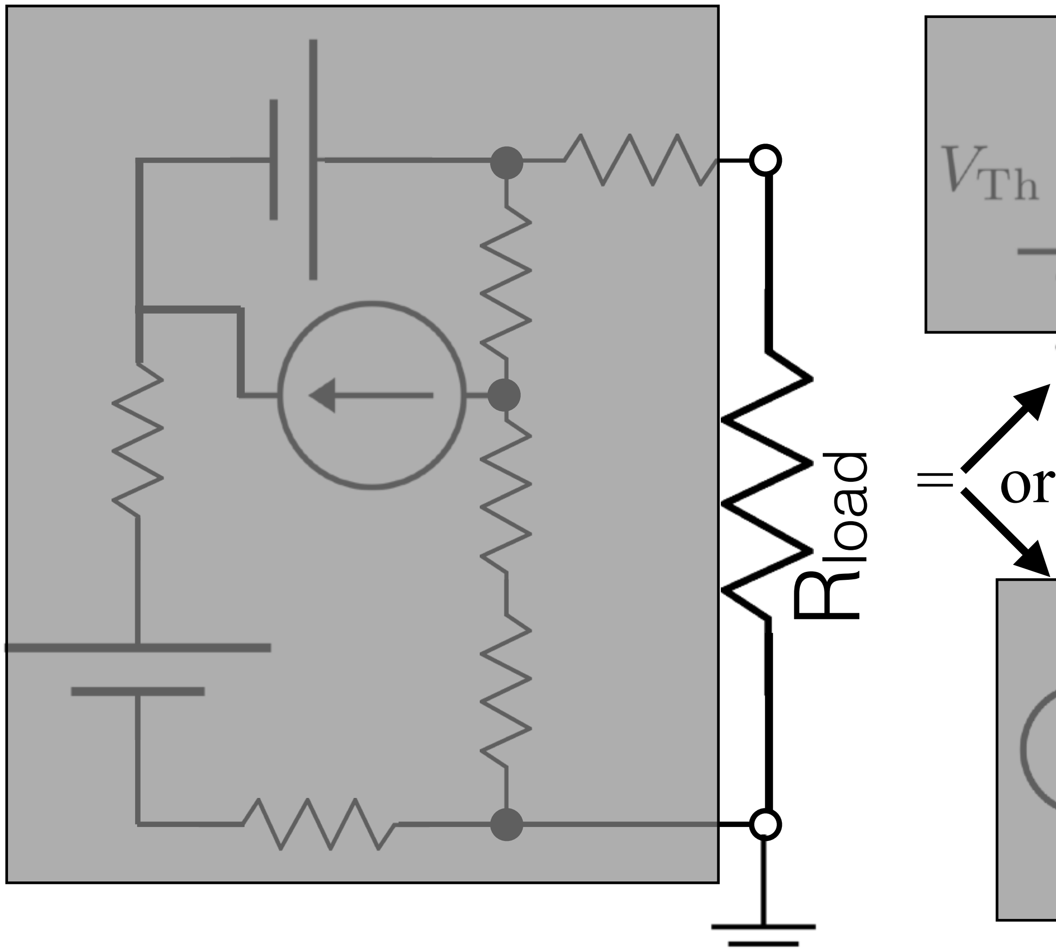
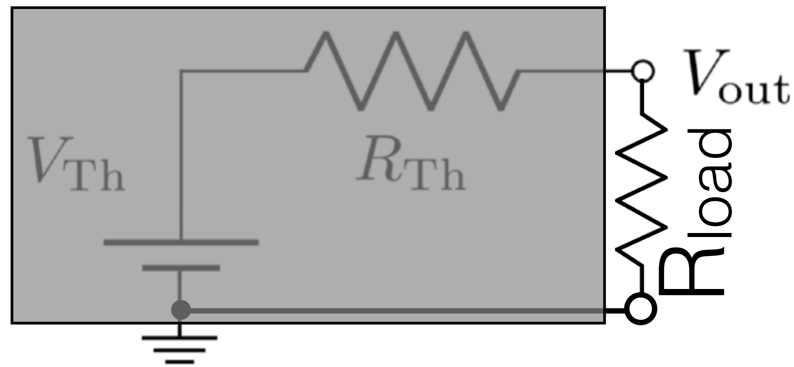
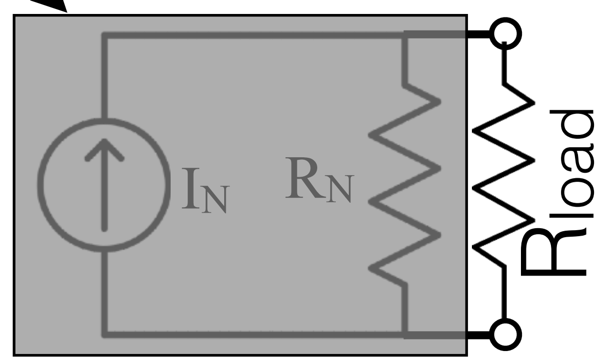


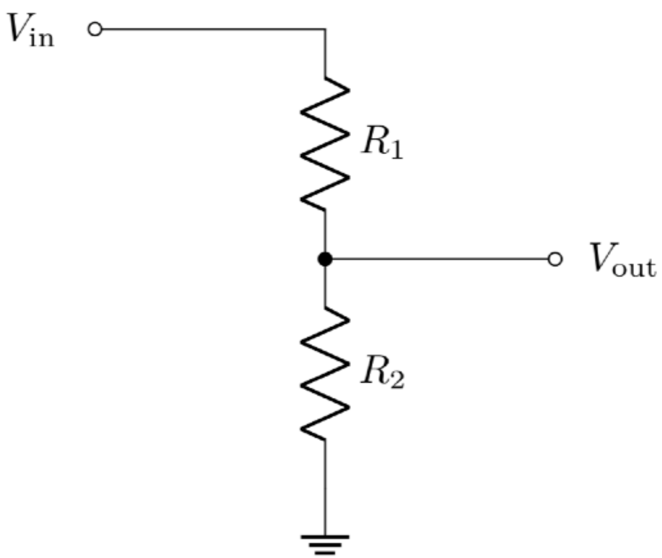




Calculate these by using linearity and extreme values of R_{Load}, ie 0 and ∞.

Voltage divider

This circuit is a voltage divider. We'll use this a lot, and generalize it.



$$V_{out} = IR_2 = [V_{in}/(R_1+R_2)]\ R_2 = V_{in}\frac{R_2}{R_1+R_2}$$

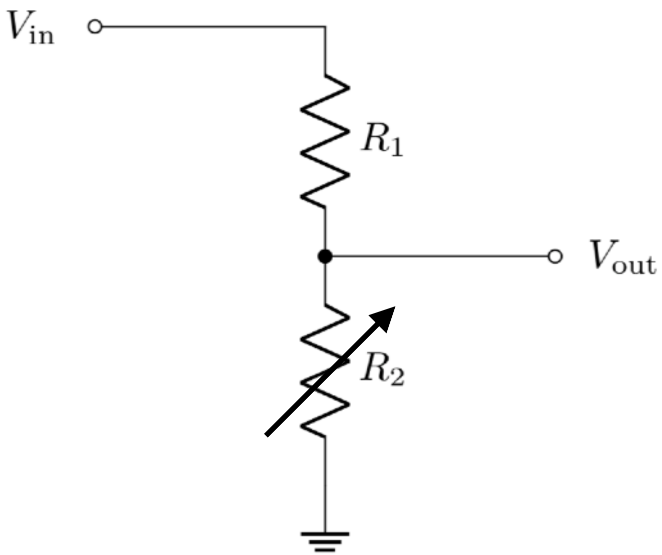
If $R_1 = R_2$ then $V_{out} = ½V_{in}$

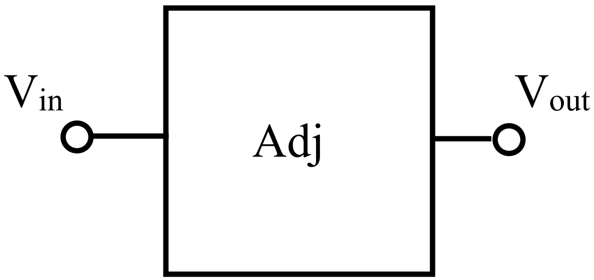
If $R_2 = 10\ R_1$ then $V_{out} \cong 0.9\ V_{in}$ which is close enough.

Trimpots used frequently for this to get an adjustable voltage.

An example circuit stage

So we could use a voltage divider as a circuit stage that outputs an adjustable voltage. This could be the dimmer switch on a lamp.





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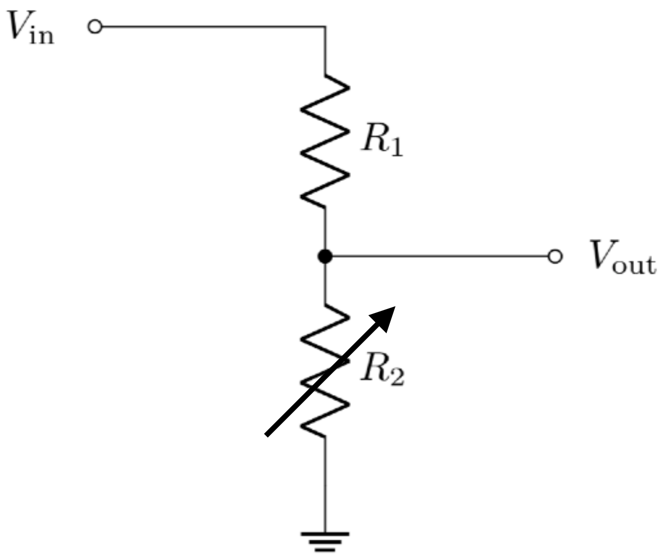

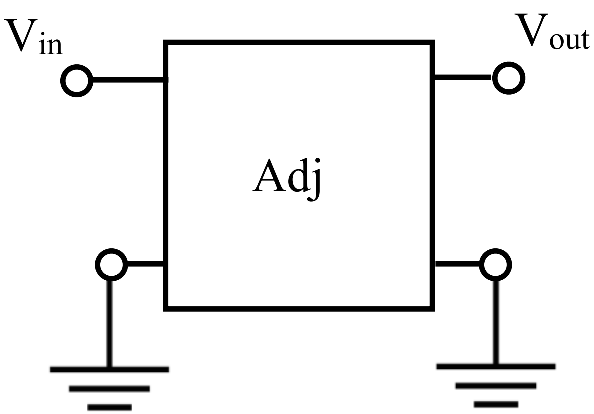

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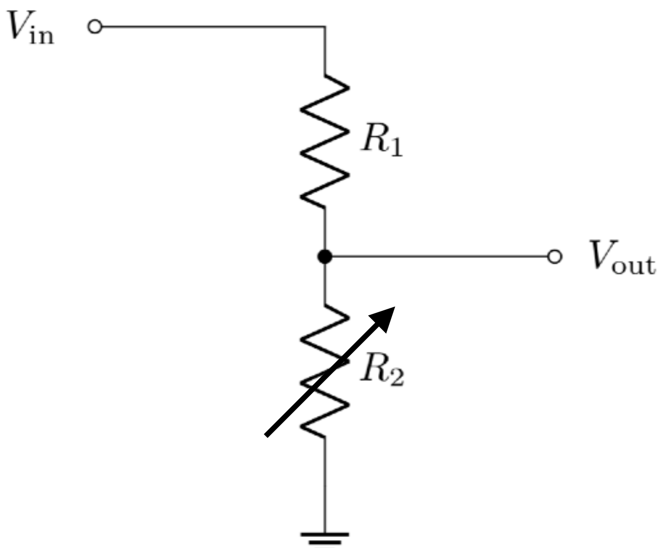

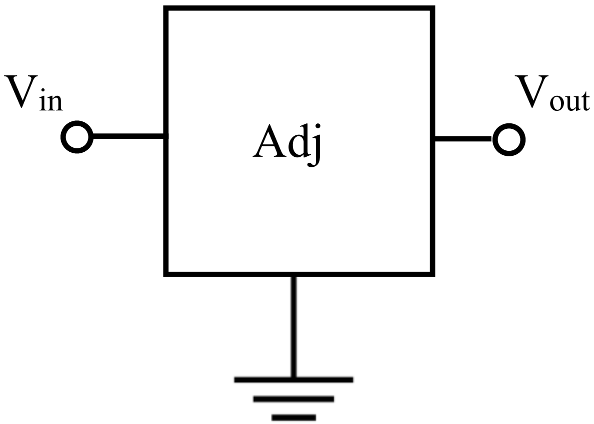

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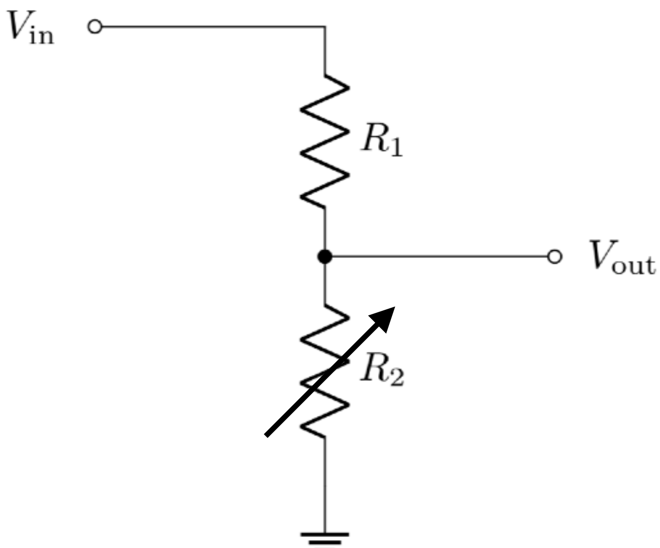

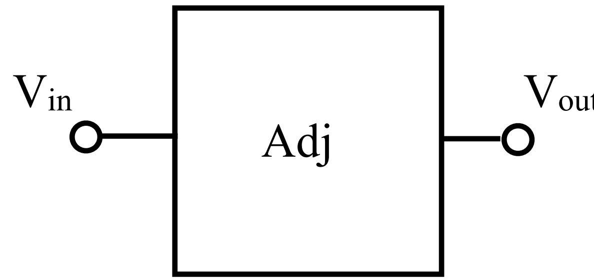

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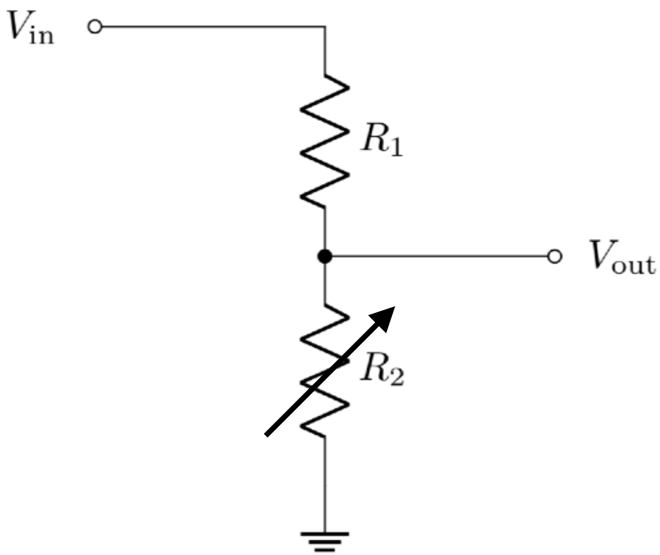

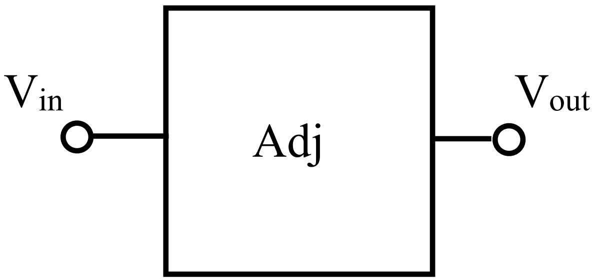

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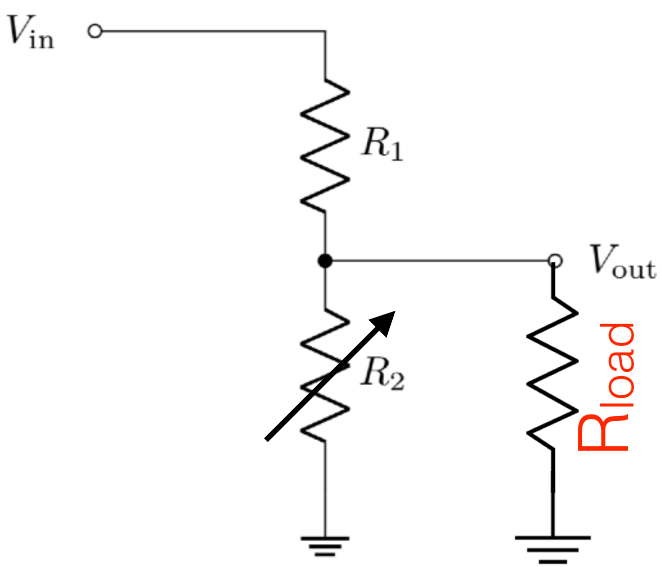

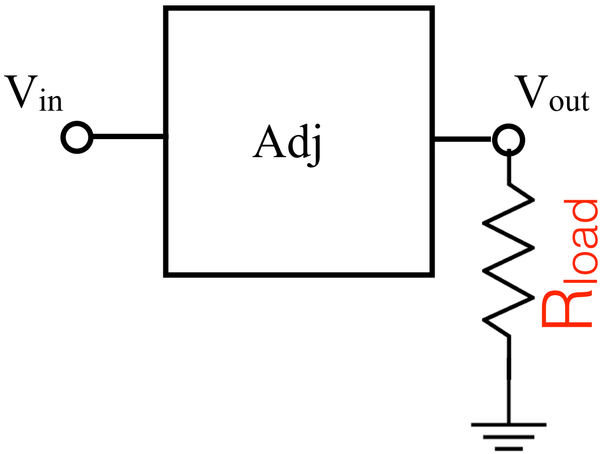

$$V_{out} = IR_2 = V_{in}\frac{R_2}{R_1+R_2} = V_{in}\frac{R_2R_{Load}/(R_{Load}+R_2)}{R_1+R_2R_{Load}/(R_{Load}+R_2)} = V_{in}\frac{R_2}{R_1+R_2+R_1R_2/R_{Load}}$$

But, suppose I now connect this stage to the next stage in my circuit. In this case, it would be a lamp. That is the load.

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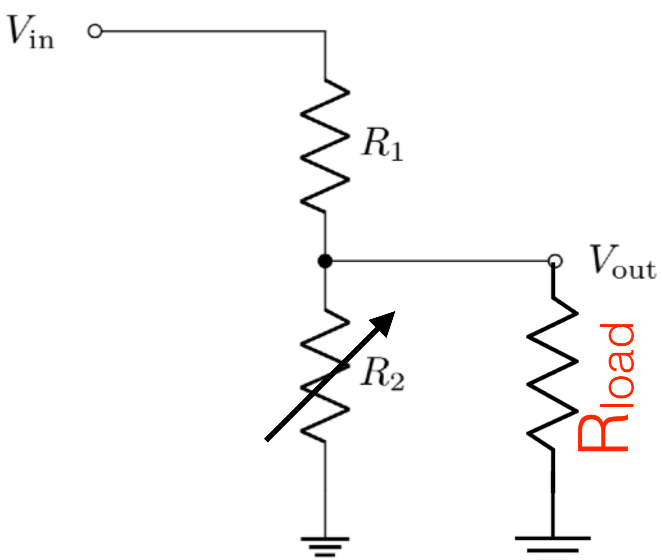

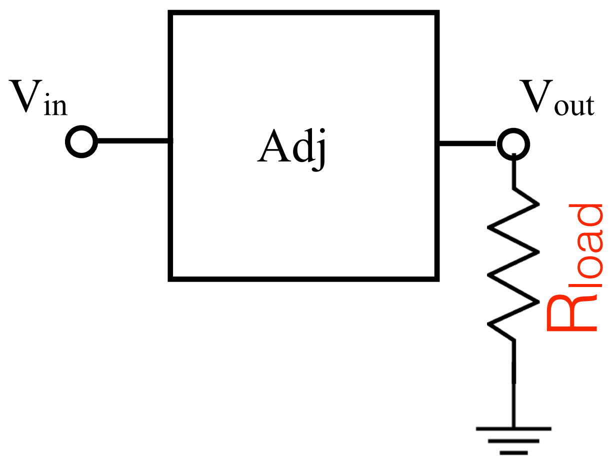

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But, suppose I now connect this stage to the next stage in my circuit.
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V_{out} is little changed by the load if $R_{Load} \gg R_1R_2/(R_1+R_2)$.

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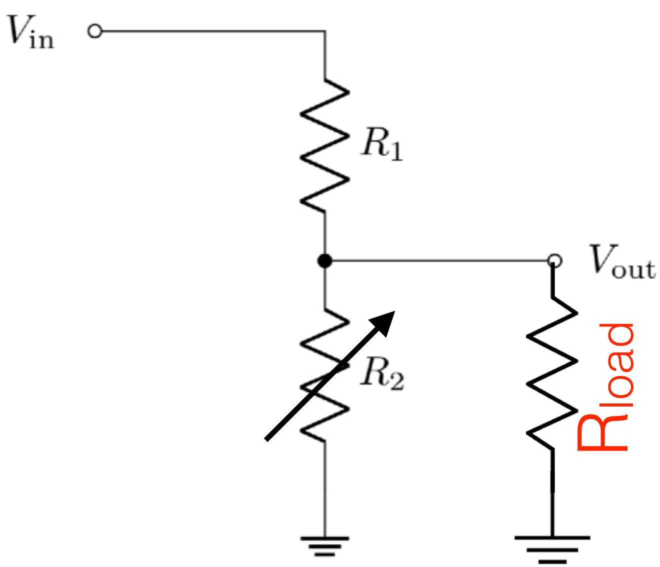

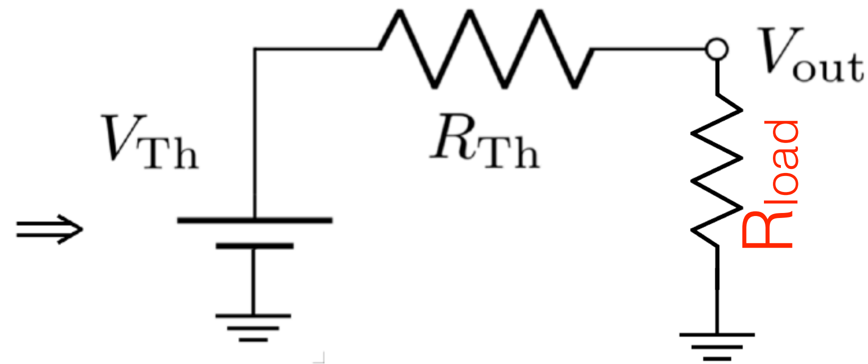

If you calculate R_{Th} you'll find
$R_{Th} = R_1R_2/(R_1+R_2)$

In general, the load causes negligible change to V_{out} wrt the stage's unloaded behavior **only if** $R_{Load} \gg R_{Th}$.

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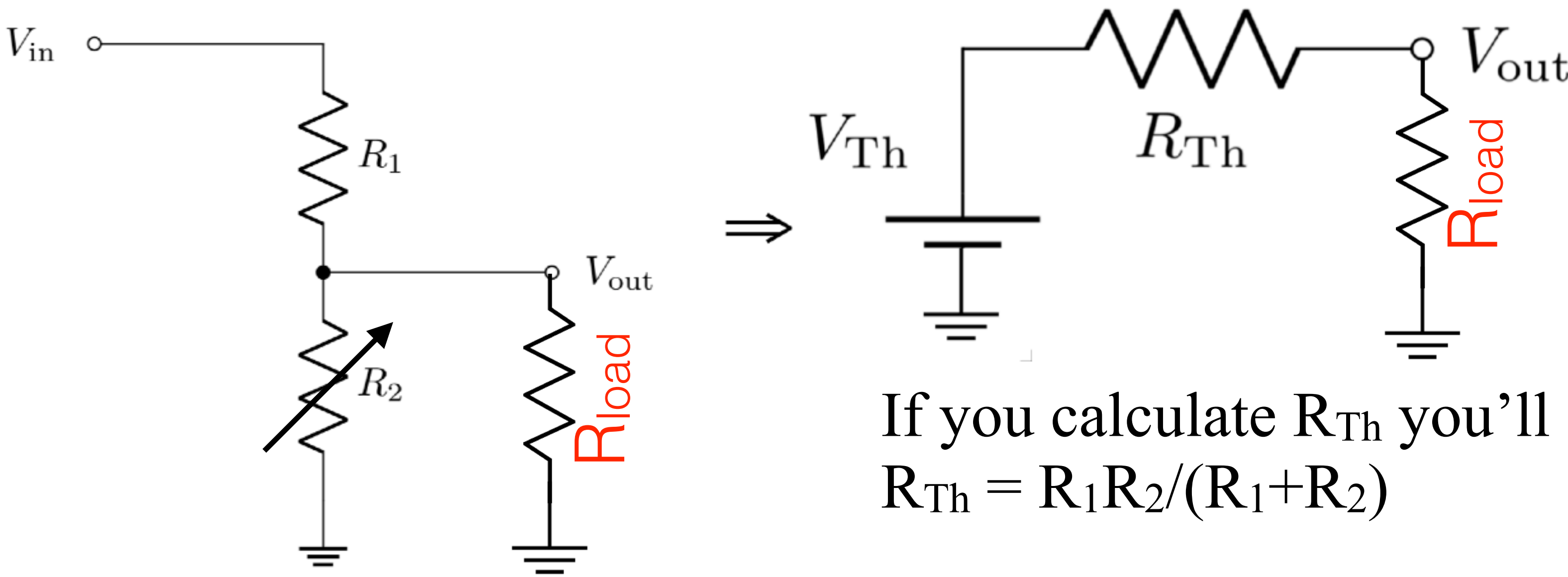

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Even more generally, we want the input resistance of the next stage to be much larger than the output resistance of this stage.

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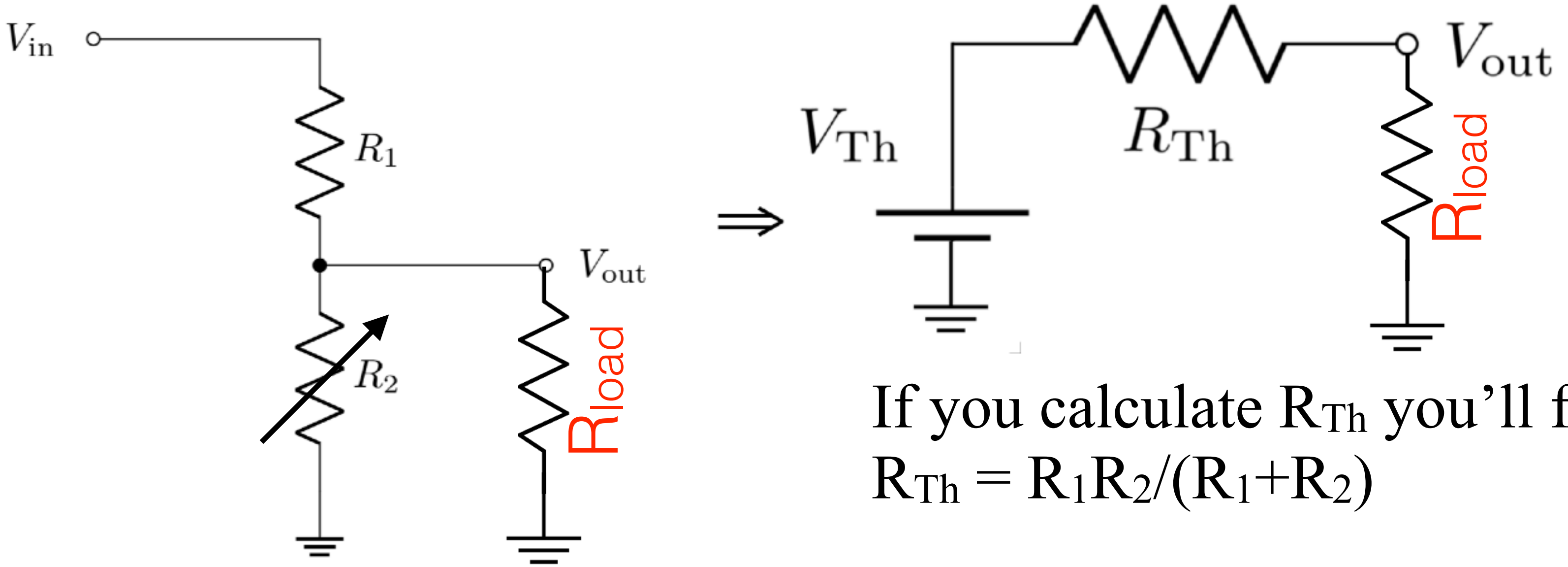

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Each stage should have large input resistance & small output resistance.

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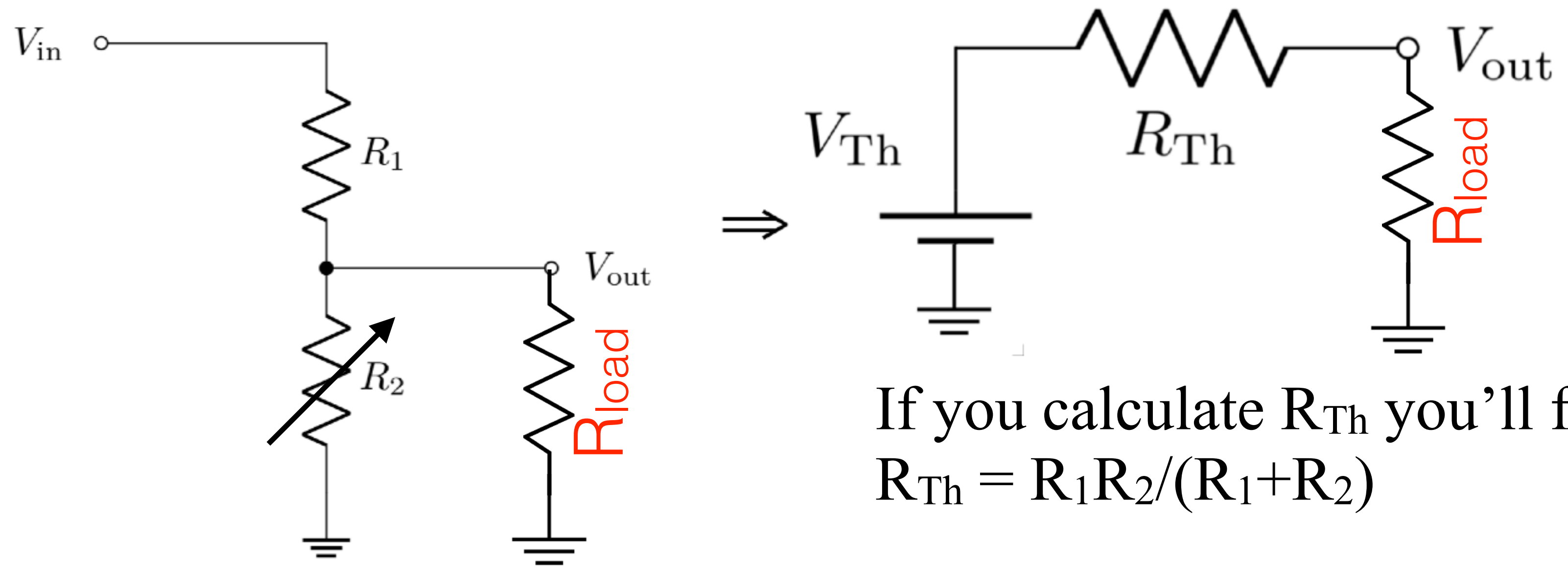

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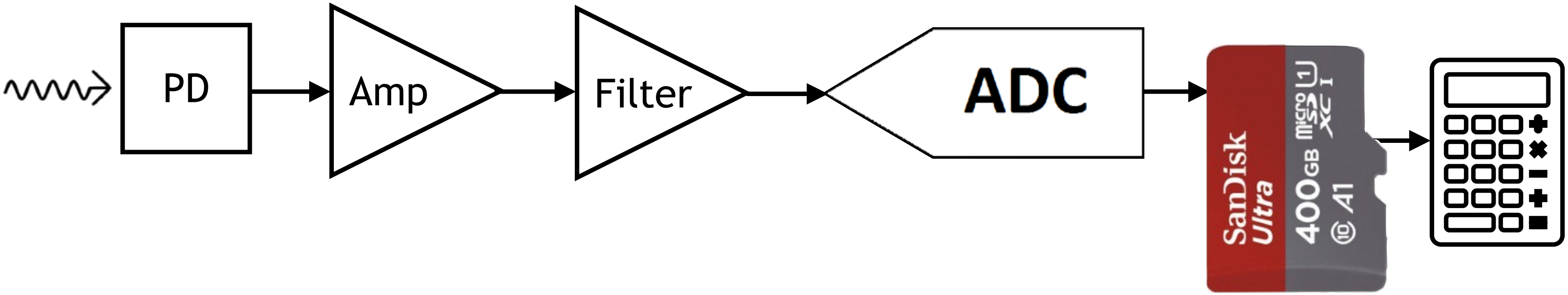
Even more generally, we want the input resistance of the next stage to be much larger than the output resistance of this stage.

Each stage should have large input <u>impedance</u> & small output <u>impedance</u>.

Chaining together stages for a measurement

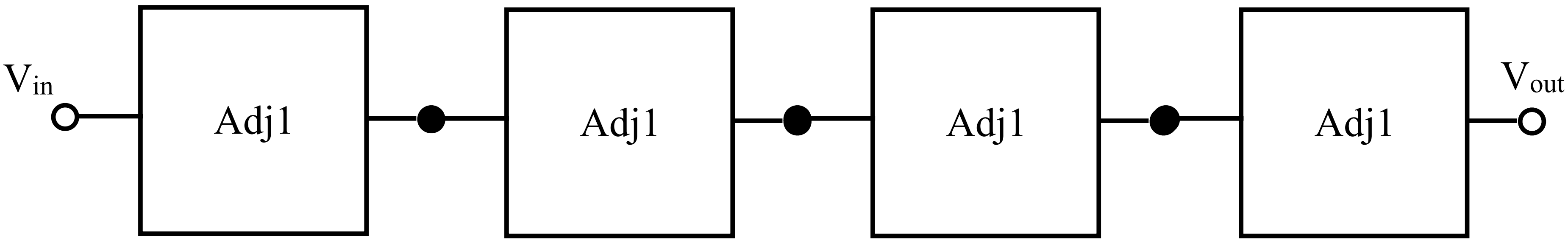
Each one of these stages should satisfy this rule, so I can chain them together without have any one affect the performance of its predecessor or successor.

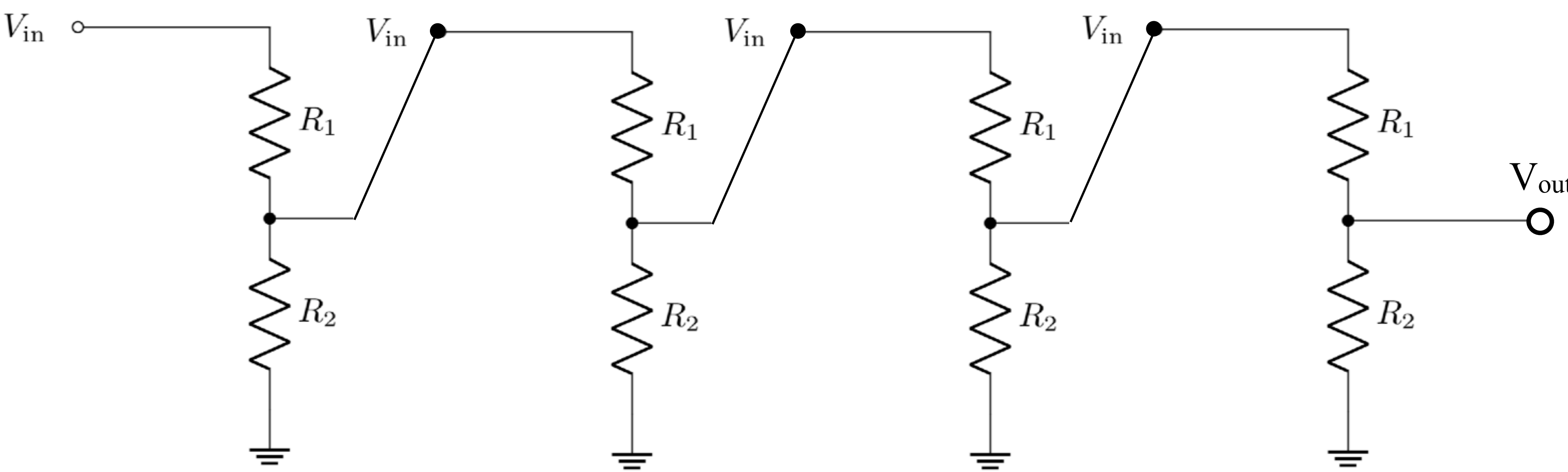


We need each stage to have high input impedance and low output impedance to easily chain together multiple stages.

x10 rule

That looks hard with multiple stages.





I'd need each stage to have resistors 10 times as big as previous stage.

That consumes high power in the early stages; $P = V^2/R$.

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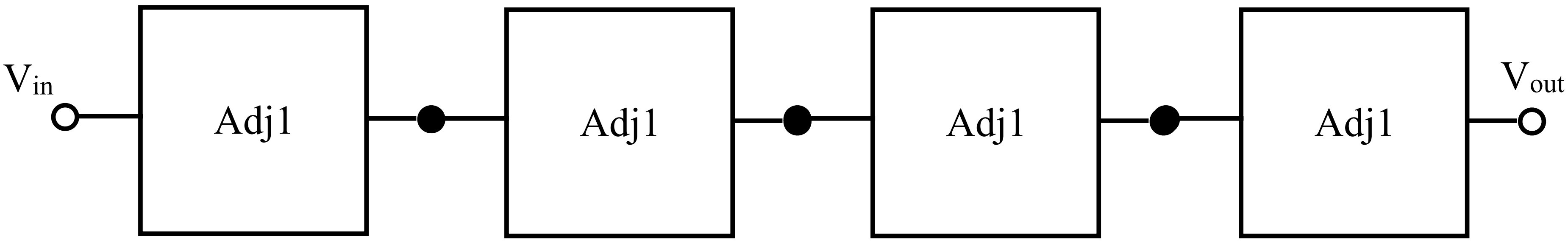
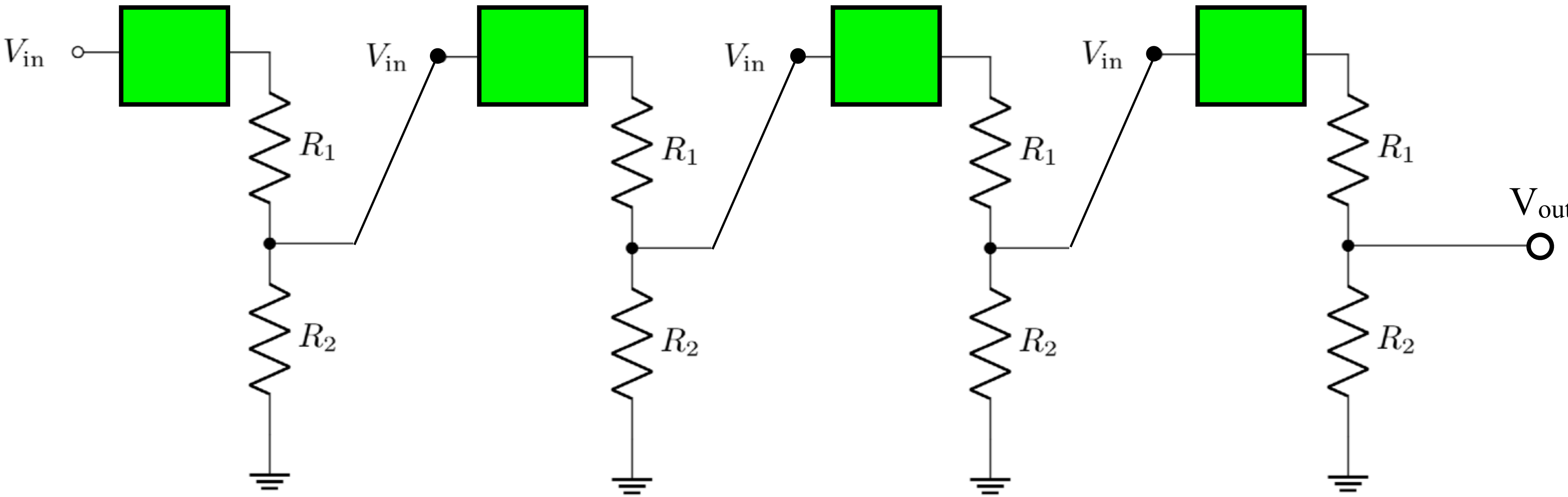




We need an "electromagical" impedance booster at the input of each stage
We'll see how to do this soon, with a transistor.

Input and output impedance

Rule for combining stages is that each stage should have high input impedance and low output impedance (resistance).

How do we calculate these, and how do they differ?

Use $R \equiv V/I$, or better $R \equiv \Delta V/\Delta I$.

To increase the voltage by ΔV how much more current, ΔI, has to flow.

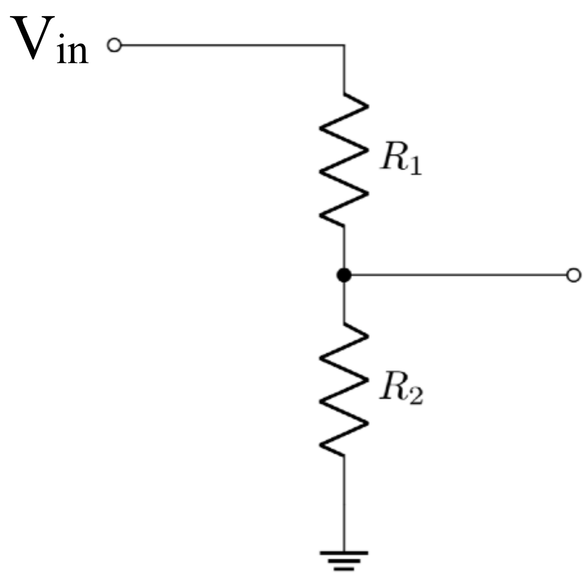
Input impedance

Use R≡V/I, or better R≡ΔV/ΔI. Increase V_{in} by ΔV; then how much more current, ΔI_{in}, flows into the input.

$V_{in} \rightarrow V_{in} + \Delta V$

causes

$I_{in} \rightarrow I_{in} + \Delta I.$



$I_{in} = V_{in}/(R_1+R_2)$

$\Delta I_{in} = \Delta V_{in}/(R_1+R_2)$

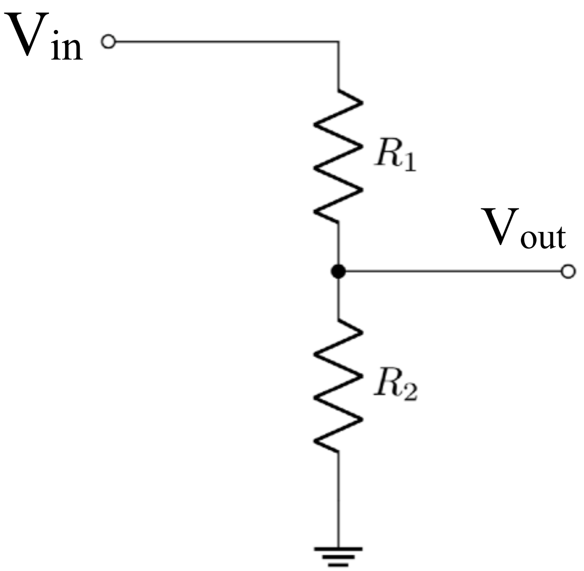
$\Delta V_{in}/\Delta I_{in} = R_1+R_2$

$R_{in} = R_1+R_2$

This is the two in series.

Output impedance

Use R≡V/I, or better R≡ΔV/ΔI. Increase V_{out} by ΔV; then how much more current, ΔI_{out}, has to flow into the output.



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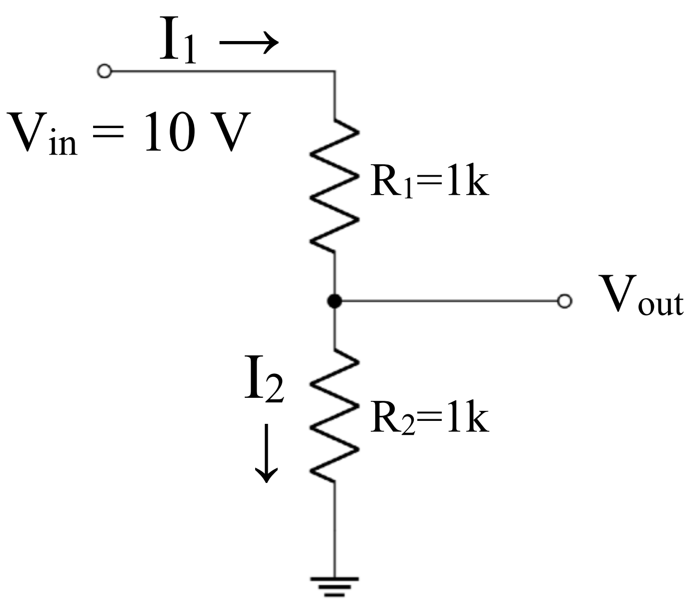
Use $R \equiv V/I$, or better $R \equiv \Delta V/\Delta I$. Increase V_{out} by ΔV; then how much more current, ΔI_{out}, has to flow into the output.

Illustrate with a numeric example, forcing V_{out}.

Find how changing $V_{out} \rightarrow V_{out} + \Delta V$ changes $I_{out} \rightarrow I_{out} + \Delta I$.

What are I_1, I_2, and V_{out}?



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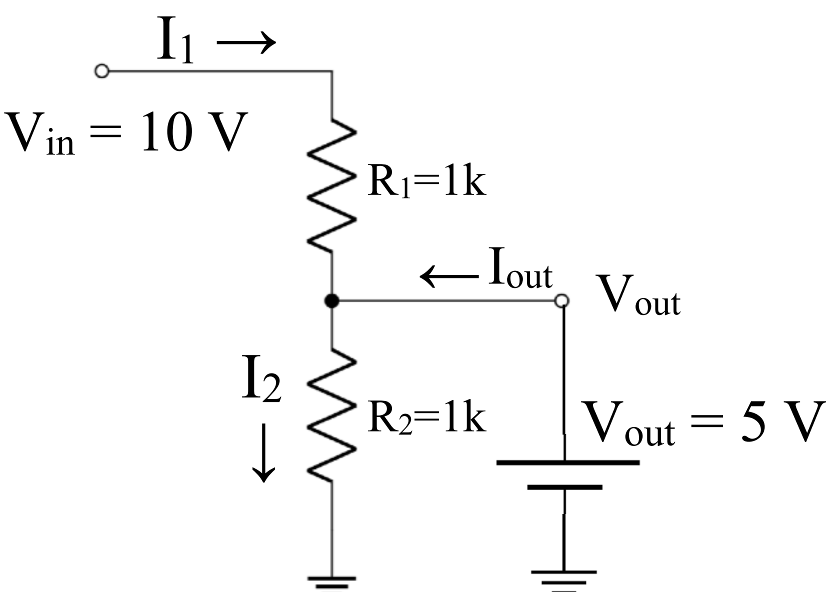


What are I_1, I_2, and I_{out}?

I_{out}=0, and $I_1 = I_2 = 10/2000$=5mA.

Change V_{out} to 6 V, i.e. ΔV=1,

now what are I_1, I_2, and I_{out}?

I_{out} is no longer 0. Find it from $I_2 = I_1 + I_{out}$.

Calculate I_1 & I_2 using Ohm's law.

$I_1 R_1 = 10\text{-}6 = 4$, so $I_1 = 4$ mA.

$I_2 R_2 = 6\text{-}0 = 6$, so $I_2 = 6$ mA.

$I_{out} = 2$ mA, $\Delta V/\Delta I = 1\text{V}/2$ mA $= 500\ \Omega$.

This is $R_1 \| R_2$.

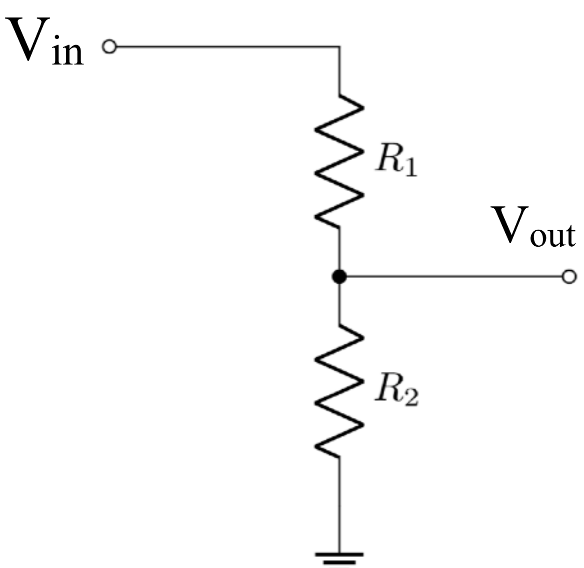
Input and output impedance

The formal definition is $R=\Delta V/\Delta I$, but we can use a simpler approach, illustrated by the example, where we found $R_{in} = R_1+R_2$ & $R_{out} = R_1 \| R_2$

R_{in} = effective resistance looking into input.

R_{out} = effective resistance looking into output.

Trace current path from input or output terminal to all fixed voltage points.



R_{in} = one path through R_1 then R_2 in series.

$R_{in} = R_1 + R_2$.

Input and output impedance

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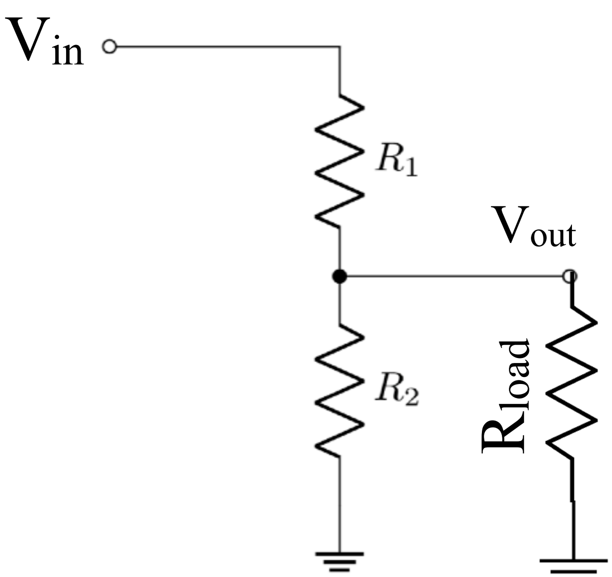

R_{in} = one path through R_1 then two ∥ paths.

$R_{in} = R_1 + R_2 \| R_{Load} \cong R_1 + R_2$, if R_{Load} is big.

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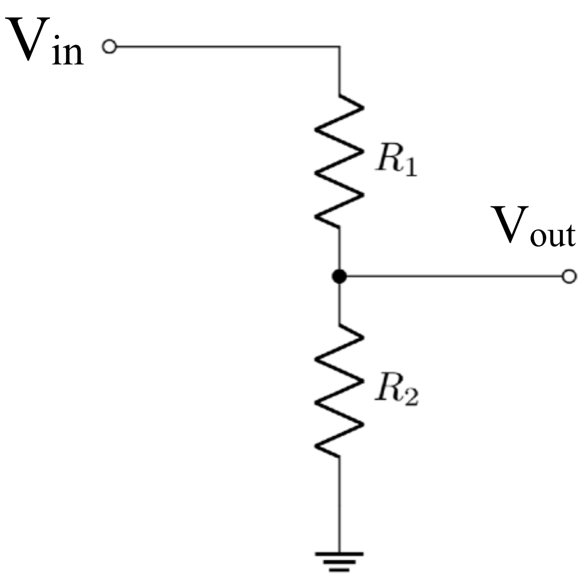

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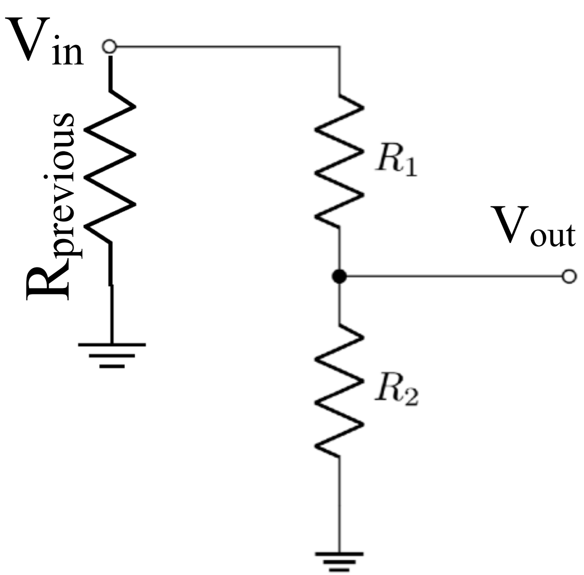

R_{out} = path 1 or path 2

R_{out} = path 1 ‖ path 2

$R_{out} = R_1 \| R_2$.

$R_{out} = (R_1 + R_{previous}) \| R_2$.

$R_{out} \cong R_1 \| R_2$ if $R_{previous}$ is small.

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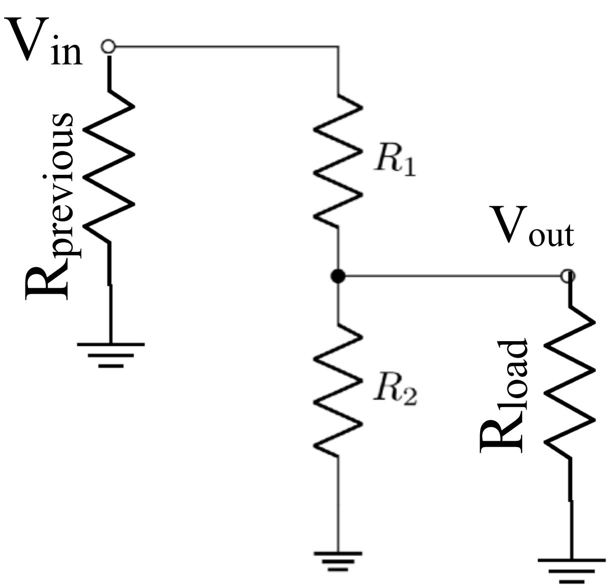
Trace current path from input or output terminal to all fixed voltage points.



$R_{in} \cong R_1 + R_2$, if R_{Load} is big.

$R_{out} \cong R_1 \| R_2$ if $R_{previous}$ is small.

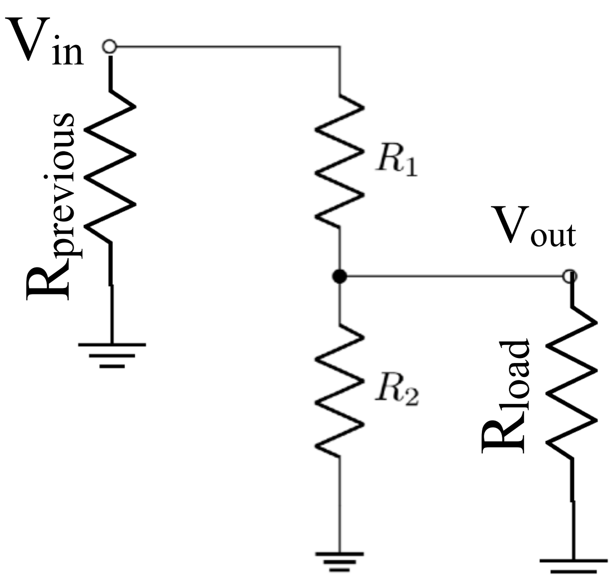
Input and output impedance

The formal definition is $R=\Delta V/\Delta I$, but we can use a simpler approach, illustrated by the example, where we found $Rin = R_1+R_2$ & $Rout = R_1 \| R_2$

R_{in} = effective resistance looking into input.

R_{out} = effective resistance looking <u>into</u> output.

Trace current path from input or output terminal to all fixed voltage points.



$R_{in} \cong R_1 + R_2$, if R_{Load} is big.

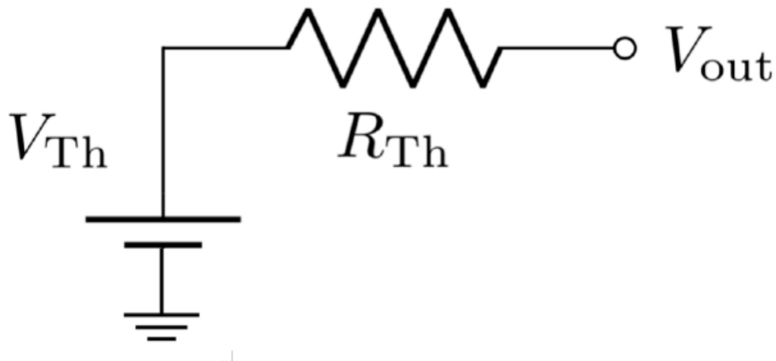
$R_{out} \cong R_1 \| R_2$ if $R_{previous}$ is small.

Design to have big $R_{Load} = R_{in}$ of next stage and small $R_{previous} = R_{out}$ of previous stage. Then they won't impact ability to get a big R_{in} and small R_{out} for this stage.

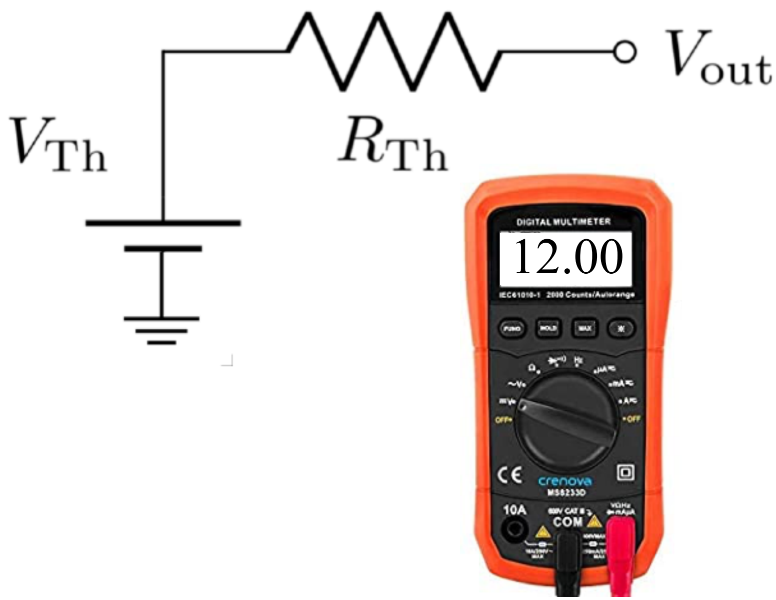
We will eventually see how to make $R_{in} \to \infty$ and $R_{out} \to 0$.

Ideal voltage and current sources

In the Thevenin equivalent circuit, we treated V_{Th} as a battery.



Batteries have a set EMF.
Ideally a 12V battery will always give 12V output. But what happens when it discharges?



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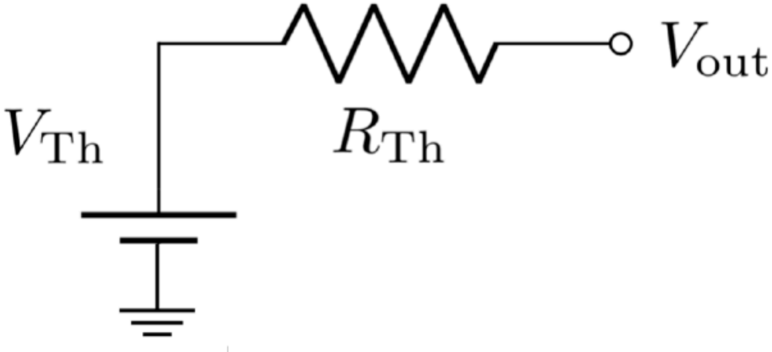

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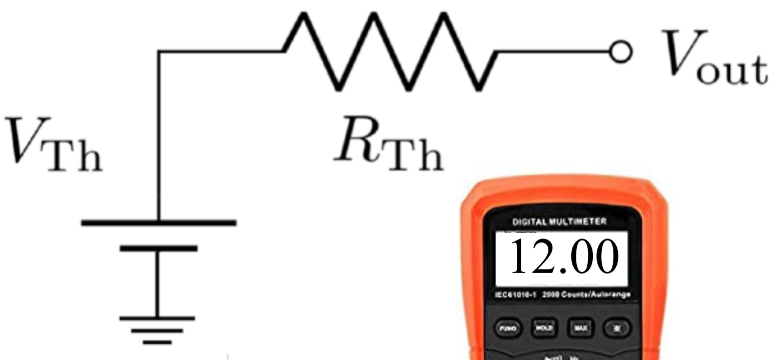

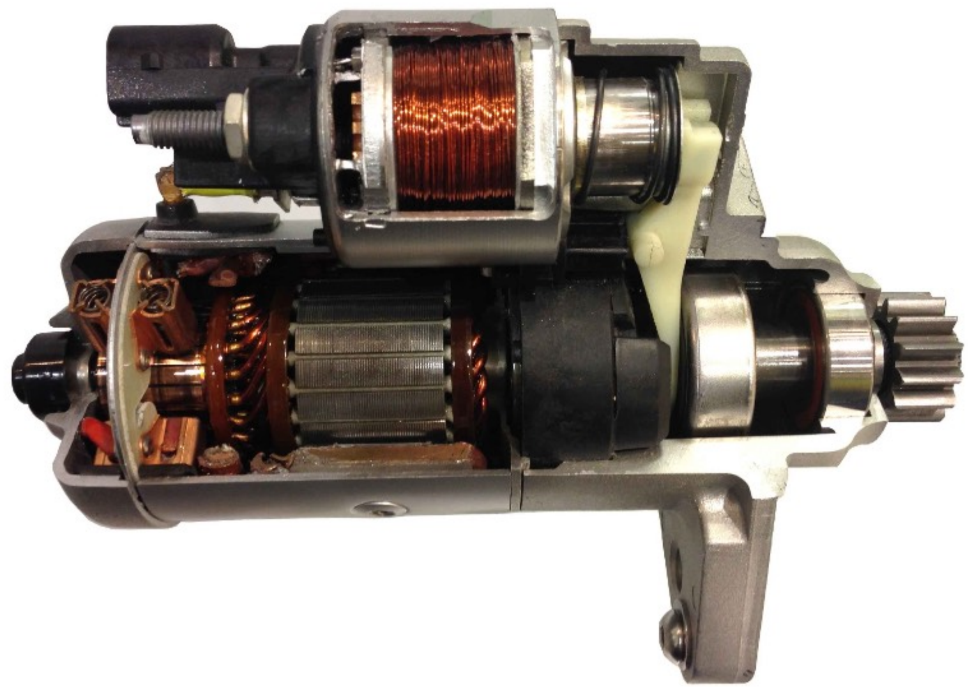

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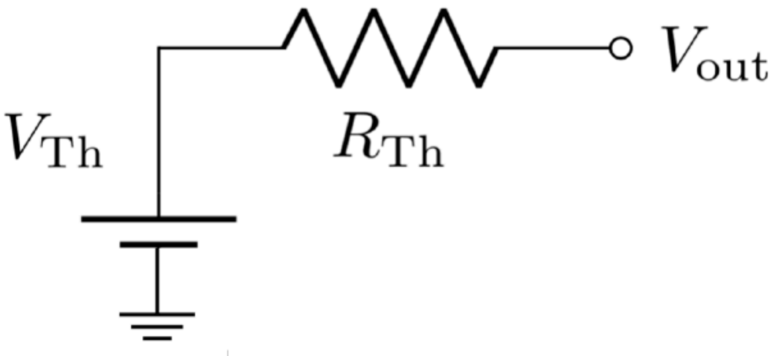

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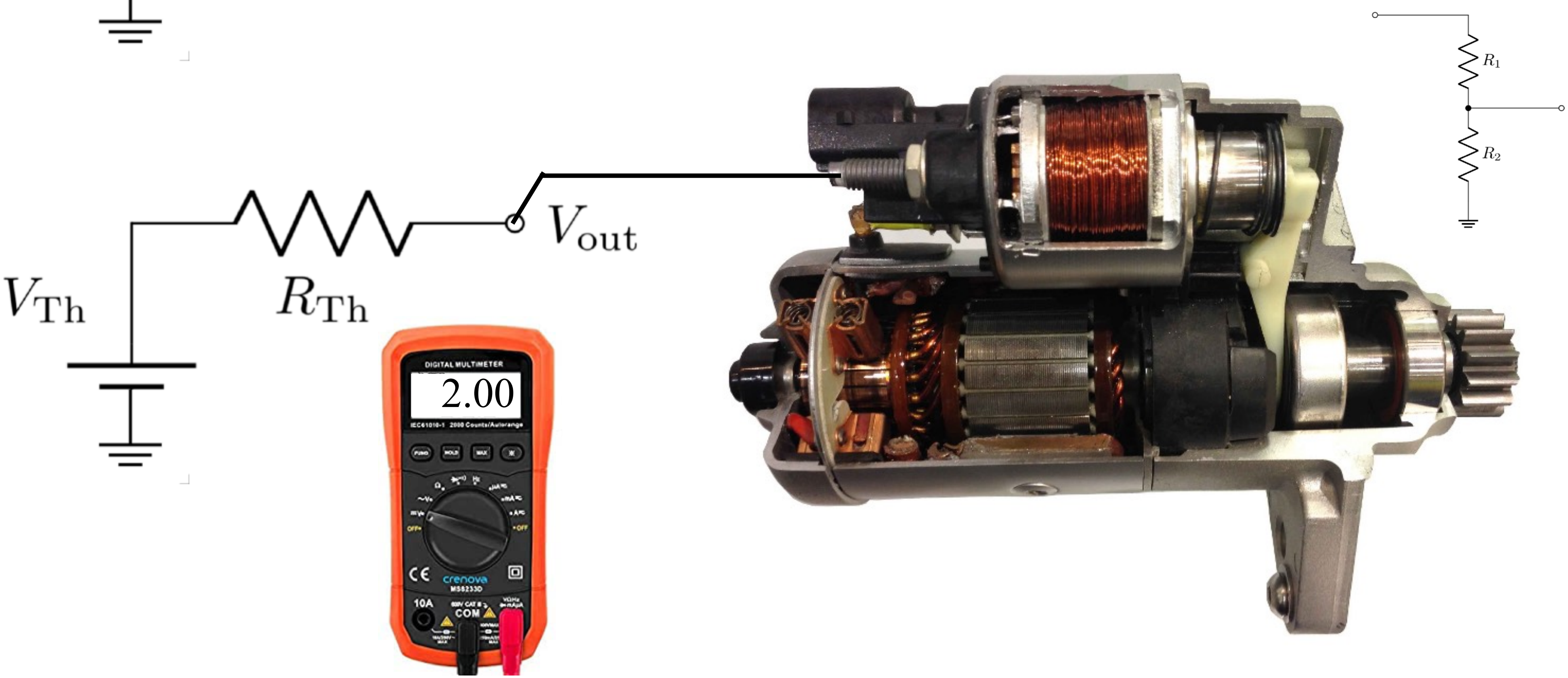

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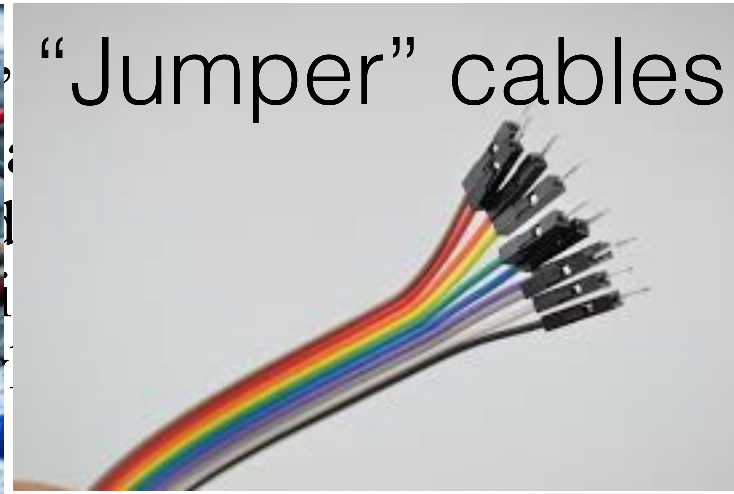
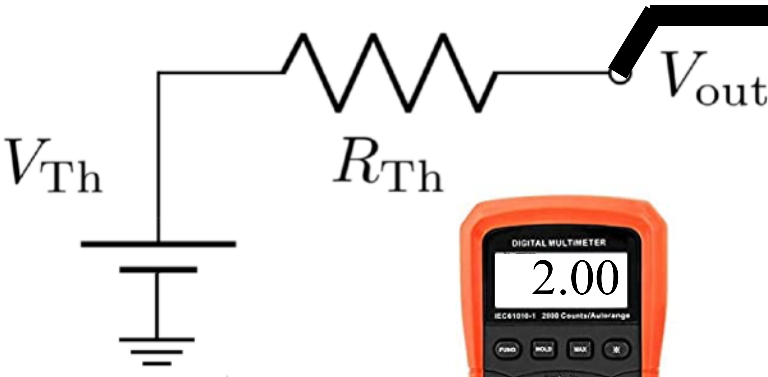
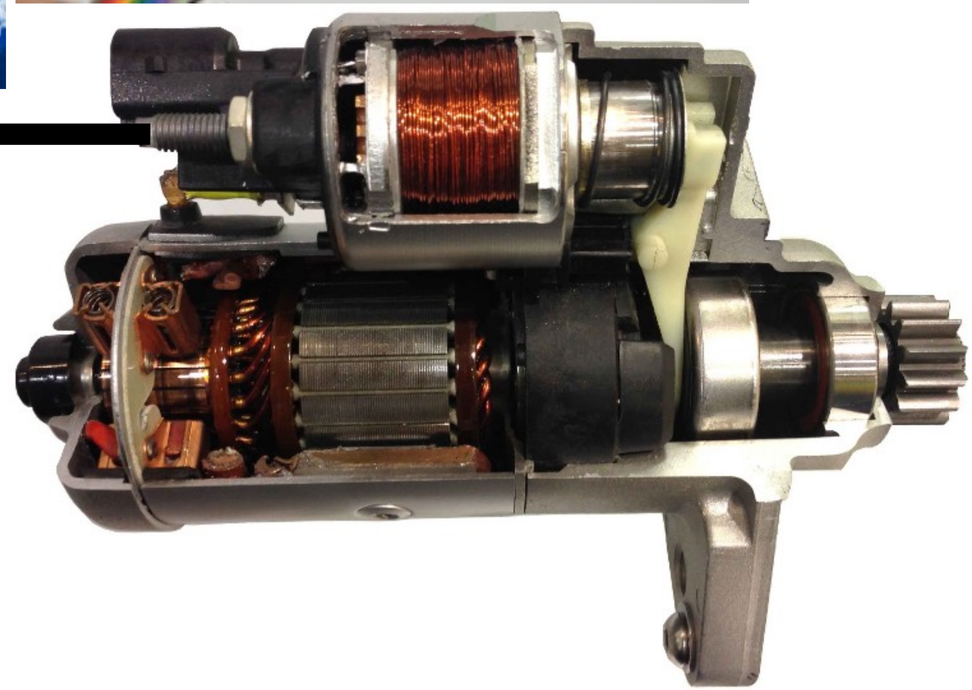




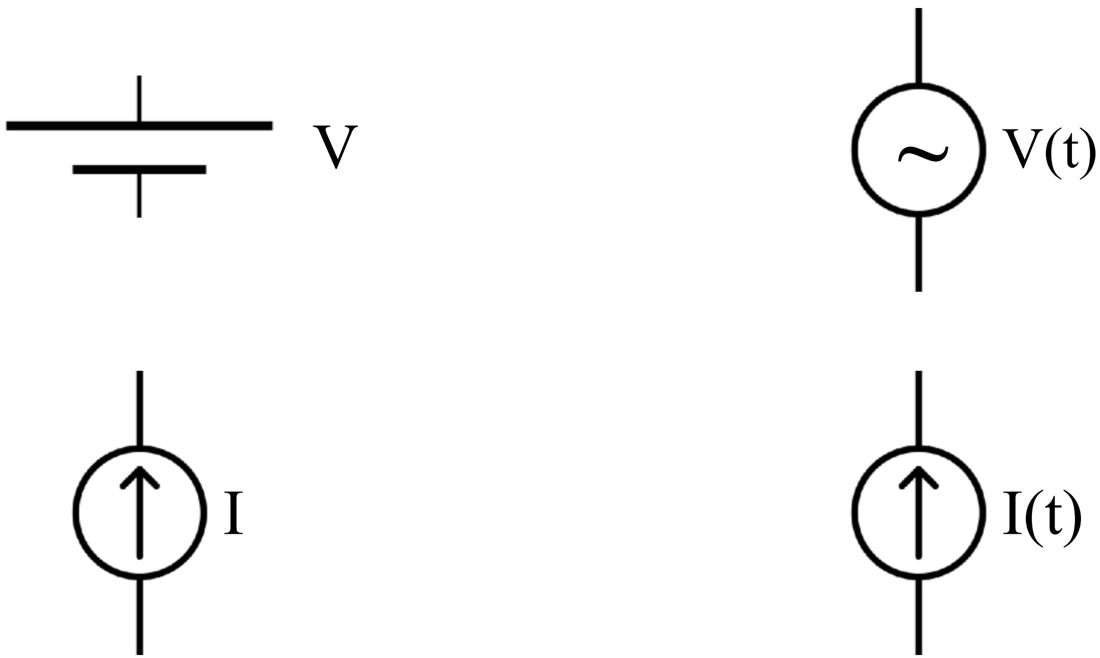


AC/DC and signals

The signals we are interested in measuring are usually time dependent, rather than some constant voltage.

Constant voltage & current sources.

Alternating voltage & current sources.

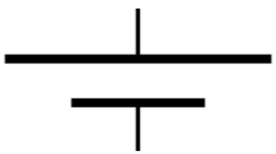


AC/DC and signals

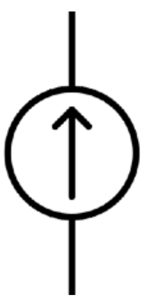
The signals we are interested in measuring are usually time dependent, rather than some constant voltage.

Constant voltage & current sources.

Direct current (DC)



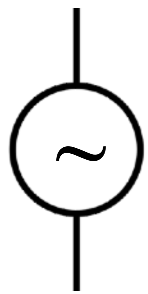
V



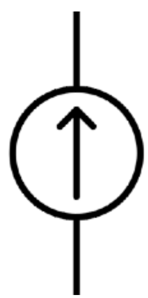
I

Alternating voltage & current sources.

Alternating current (AC)



V(t)



I(t)

AC signals can be sine waves, or anything

Sine wave:

$$V(t) = A\sin(2\pi f t) + V_0$$

$$V(t) = A\sin(\omega t) + V_0$$

Square wave

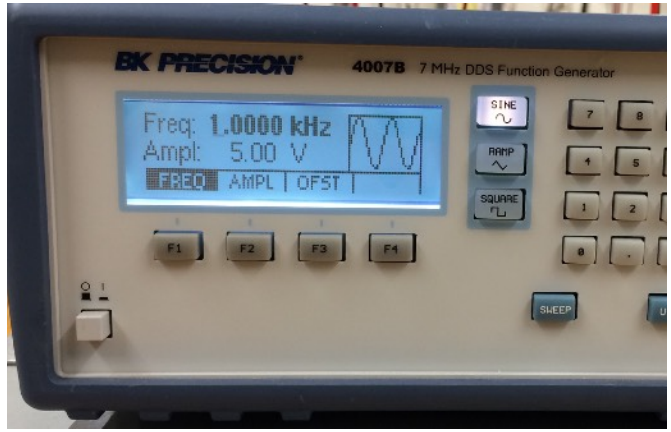
Triangle wave

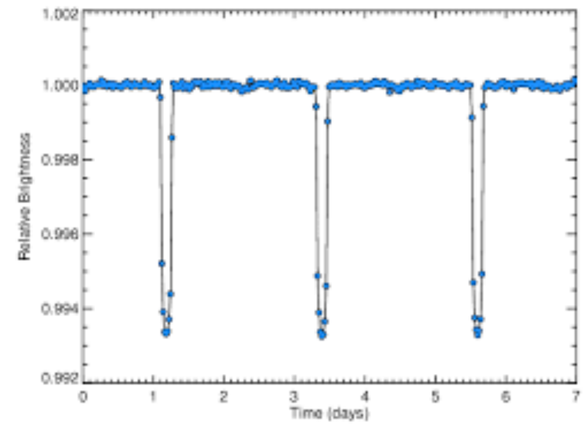
Sawtooth

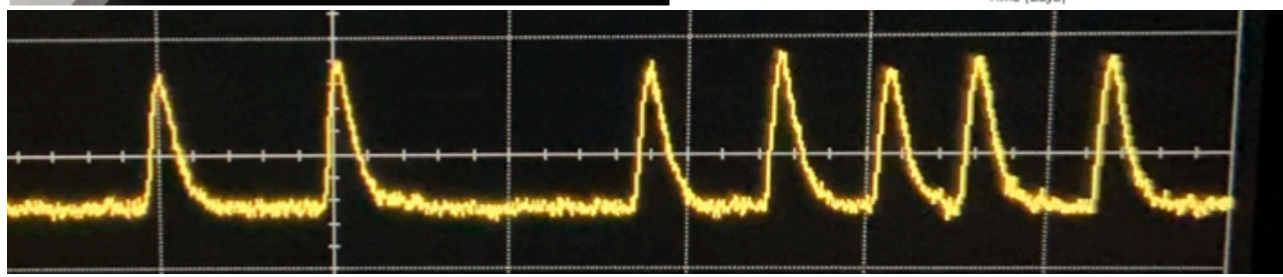
Pulse

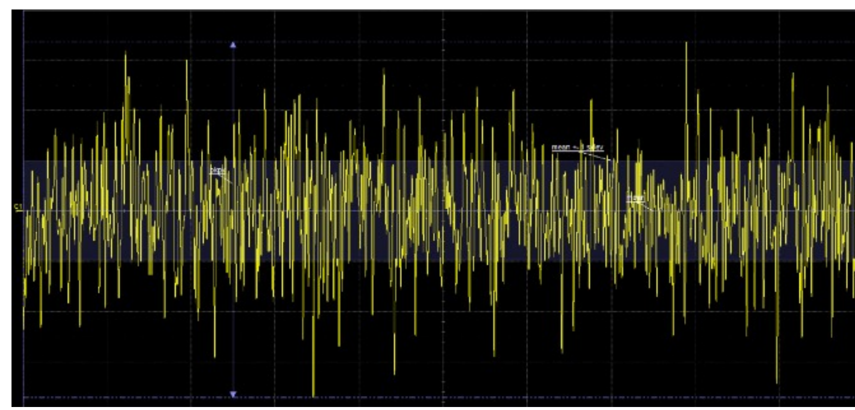
General variation

Noise (1/f noise has same power/Hz)









We'll typically deal with sine wave signals, because any wave can be treated as a Fourier sum of them.

Parameters of a sine wave

Sine wave:

$$V(t) = A\sin(2\pi f t) + V_0$$

$$V(t) = A\sin(\omega t) + V_0$$

We'll typically deal with sine wave signals, because any wave can be treated as a Fourier sum of them.

Amplitude often specified as $V_{RMS} = A/\sqrt{2}$

This is what matters most for power consumption.

$$P = IV = V^2/R$$

and average of $\sin^2$ is 1/2.

Electrical power in your house is 120 V RMS, so A=170 V.

Ratios of signals and decibels

The ratio of two AC signals is often described in decibels, which you probably learned about for sound waves.

1 dB = 20 log (A_2/A_1)

+6 dB if $A_2 = 2A_1$

+20 dB if $A_2 = 10A_1$

-20 dB if $A_2 = A_1/10$

Common to compare power levels, and $P \propto V^2$, so

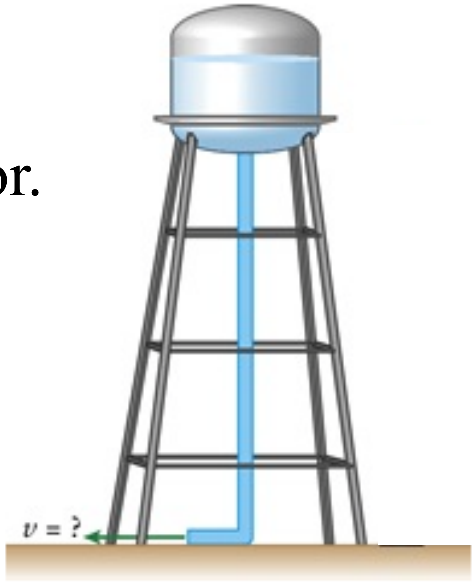
1 dB = 10 log (P_2/P_1)

-3 dB corresponds to half the power, or A/sqrt(2)

Capacitors

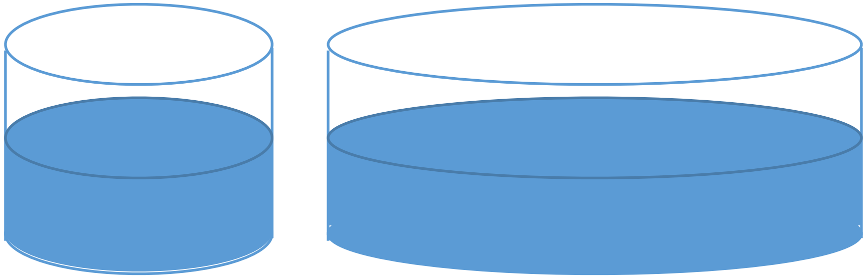
Now we get to introduce a new circuit element, which is manifestly AC.

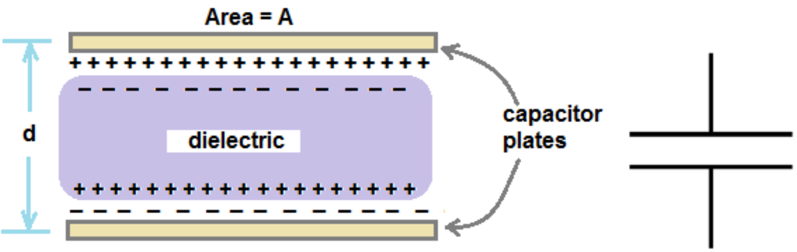
A capacitor stores charge, producing an electric field and hence energy.
V = q/C, which I remember more easily as q = CV.



In the water analogy, a water storage tank is a capacitor.

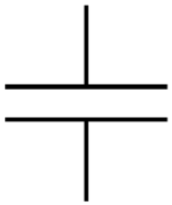
A wider tank has more capacity, ie more water (q) can be stored for the same height (V).

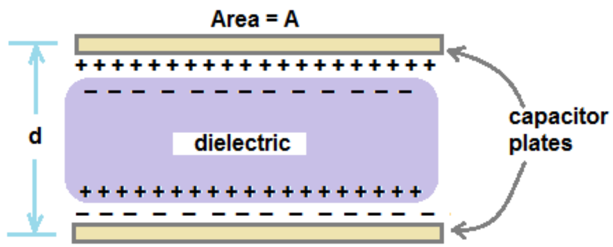




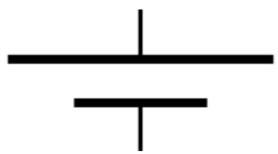
Capacitor symbol

Circuit symbol matches the visualization of a parallel plate capacitor.





Don't confuse with battery symbol.



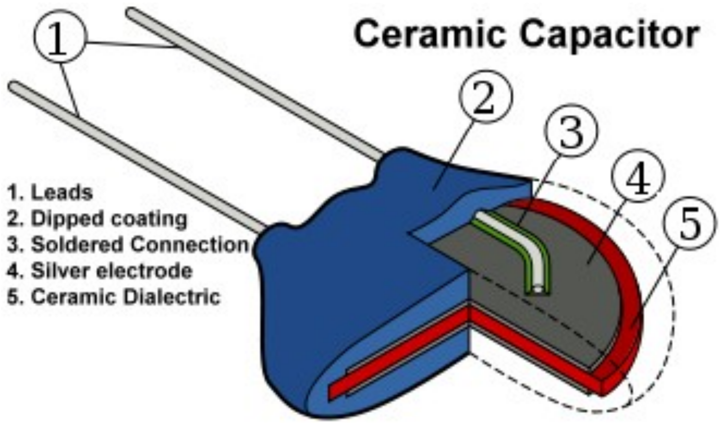
Unit is Farads, 1 F = 1 Coulomb / Volt.

Typical values are pF to mF.

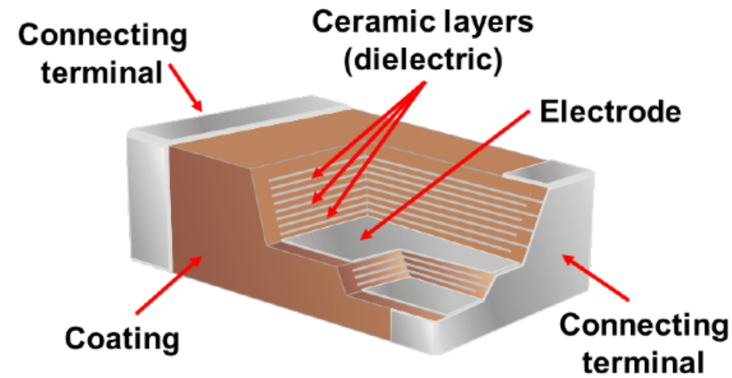
You will mostly use nF and μF.

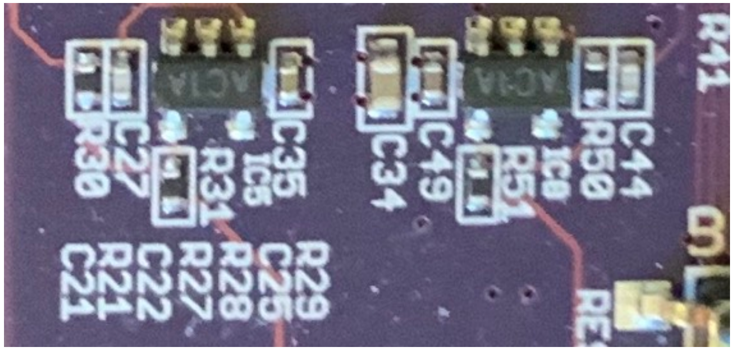
Capacitor construction

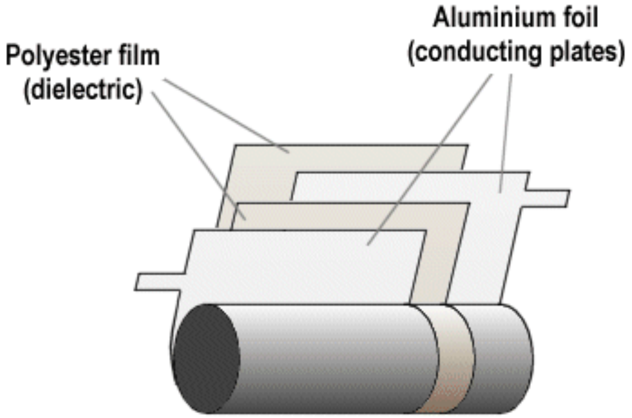
Film capacitors and ceramic are common for $C < 1\mu F$.



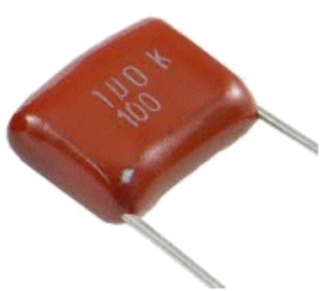
Surface Mount (SMT)







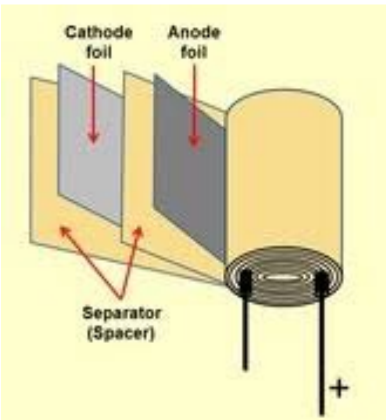
Film capacitor



Capacitor construction

Electrolytic and tantalum capacitors are common for $C > 1\mu F$.

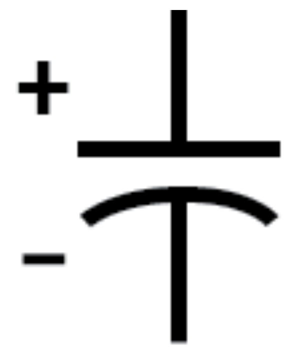
Paper separator soaked in electrolyte.



Thin oxide layer around tantalum pellet

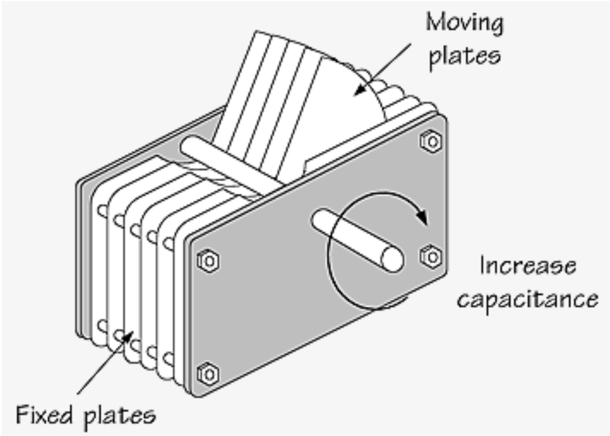
Use of an electrolyte makes these only work if polarity maintained.

So they have a different symbol.

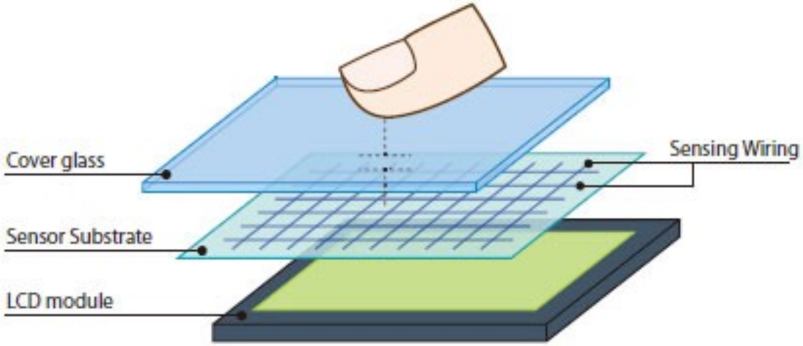


Other capacitor types

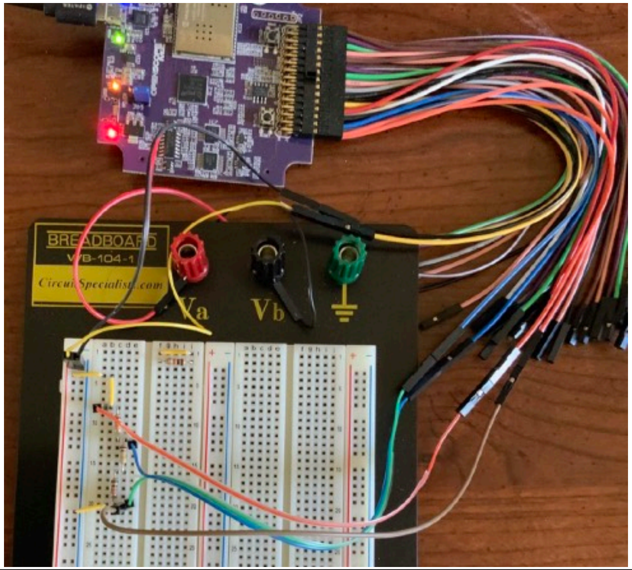
Variable capacitors can be made with C < 1 nF using moving plates.



Touch capacitors can be made from a thin elastic plastic film.



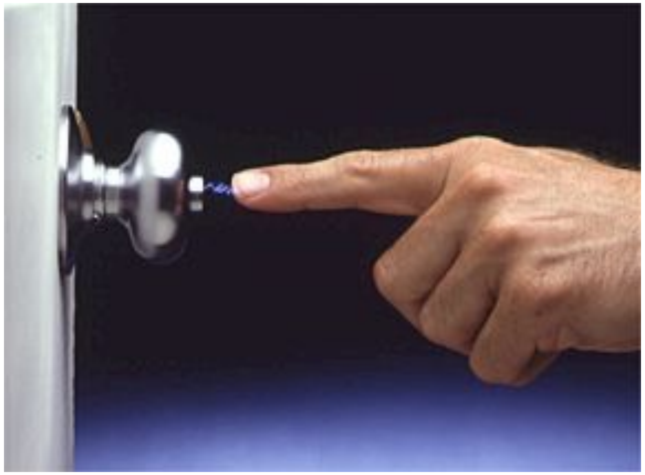
Parasitic capacitance is produced when any two conductors pass near each other. Just jostling your wires can change the capacitance in your circuit. Even moving your finger near a pair of wires.



Electrostatic discharge

Any single conductor has capacitance; the other electrode is the earth.

You have capacitance, but not very large. Adding electrostatic charge to you, by rubbing feet on a carpet, gives you are large voltage through $V = q/C$, even with a relative small charge.



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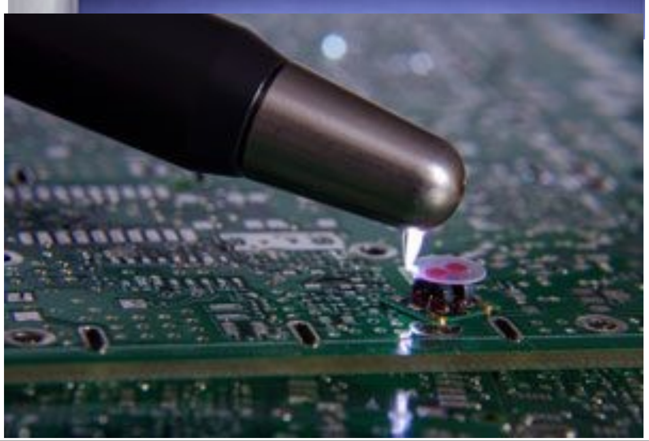
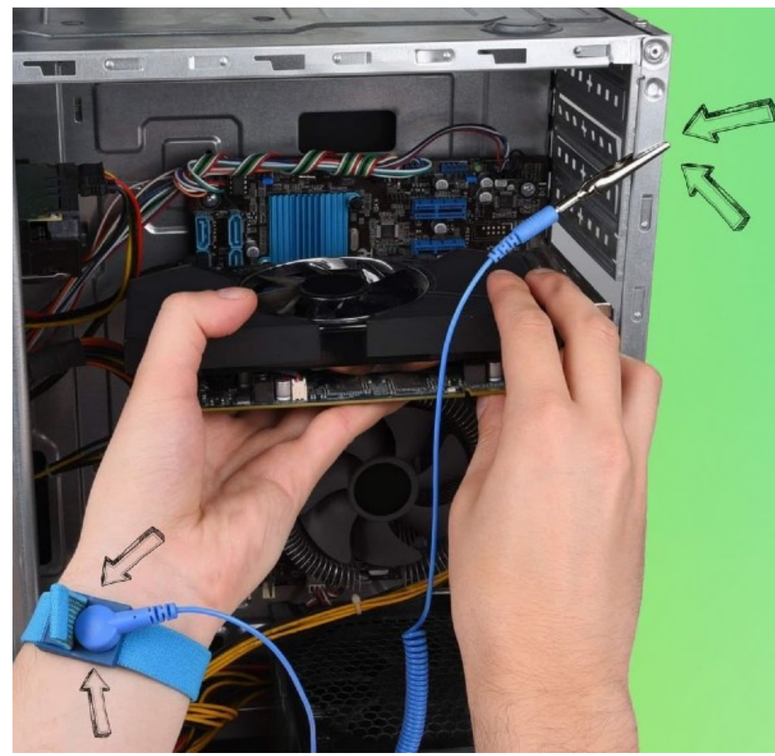
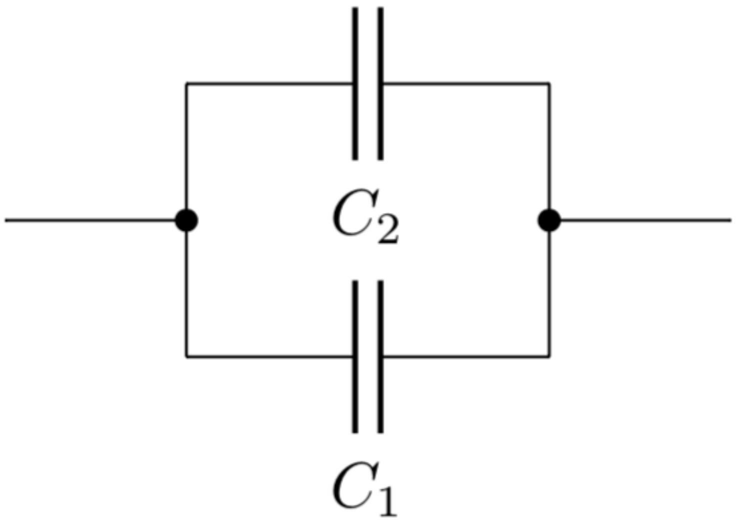


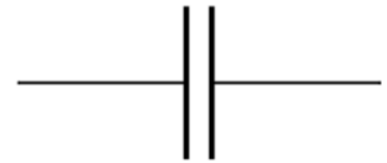


Combining capacitors

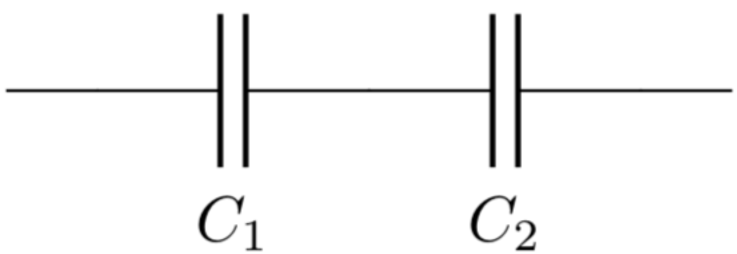
Capacitance values add in series and parallel opposite to the way resistors do. But it is intuitive: two tanks in parallel hold more water.



$=$



$$C_{\text{eff}} = C_1 + C_2$$



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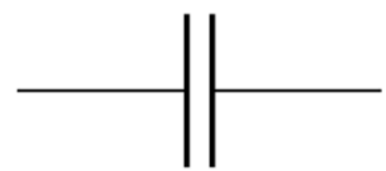
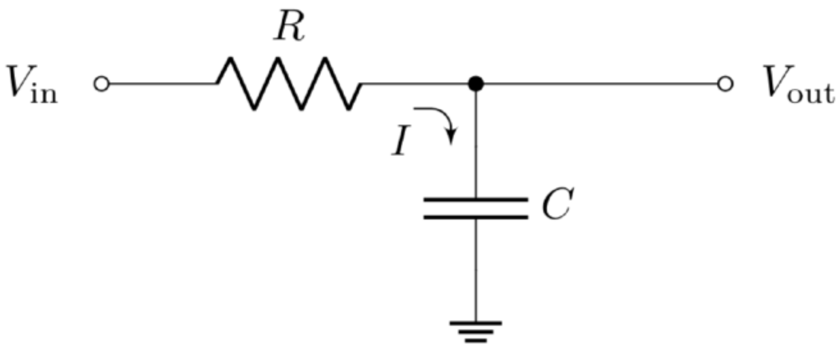


$$C_{\text{eff}}^{-1} = C_1^{-1} + C_2^{-1}$$

You can derive these from q = CV, and Kirchoff's laws.

Charging a capacitor

Suppose we charge a capacitor through a resistor with this circuit.



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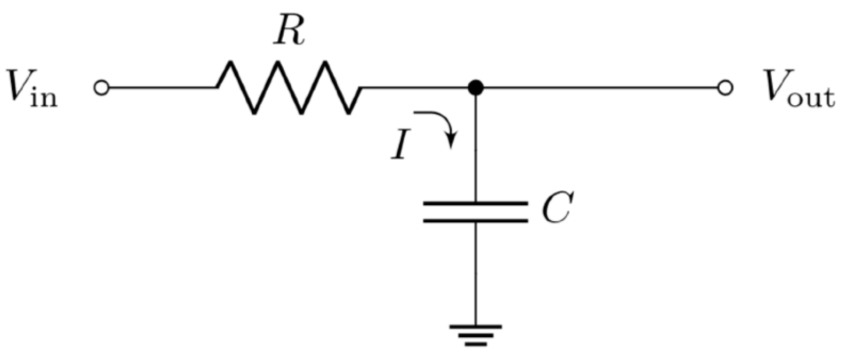

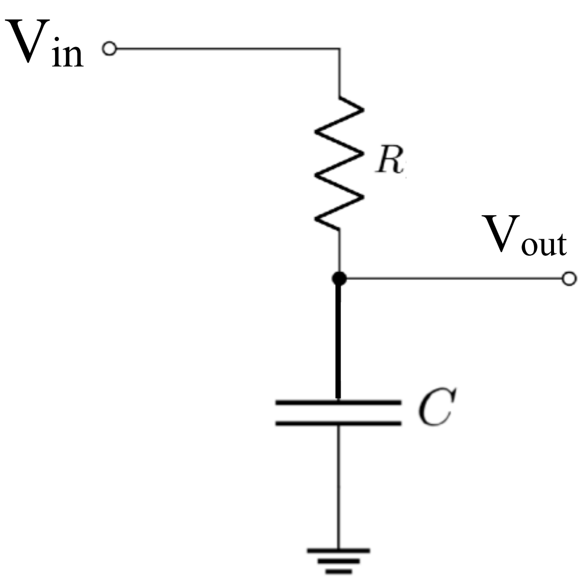

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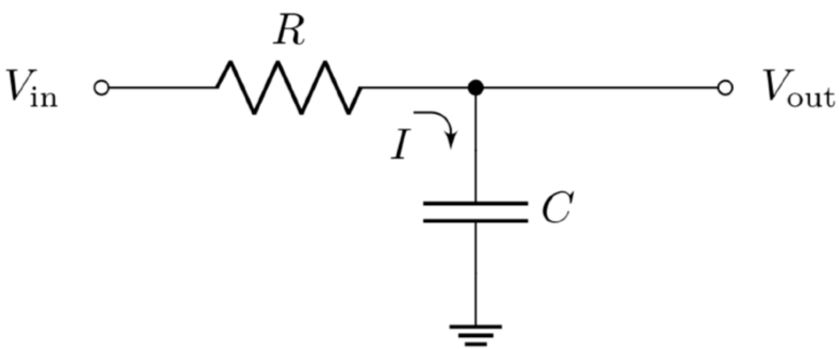
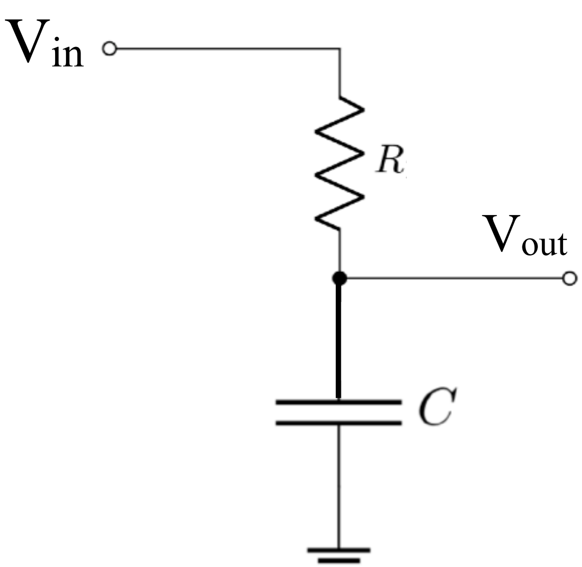


We can analyze this from

$q = CV \quad \Rightarrow \quad I = C\, dV/dt$

I *through* capacitor is related to the change in the voltage *across* the capacitor.



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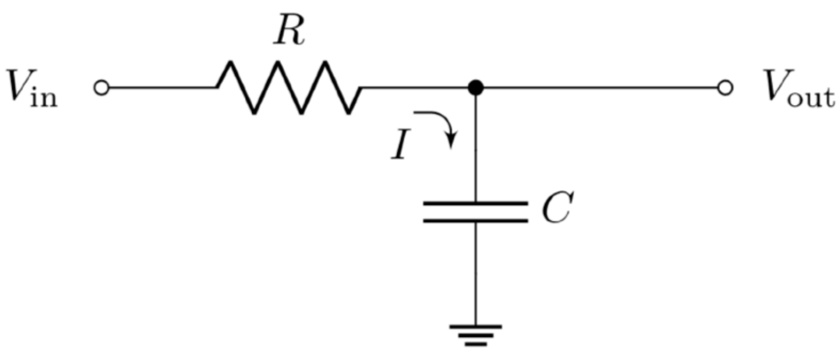

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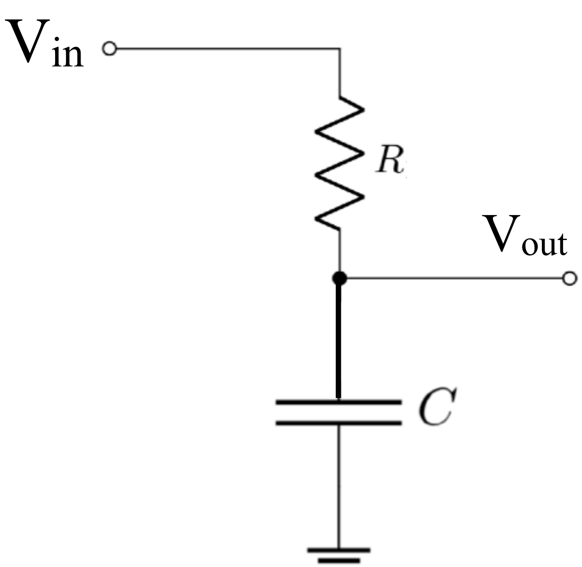
The voltage across the capacitor is V_{out}.

The current through the capacitor is whatever current is flowing through R from V_{in}, which is the voltage *dropped across* R:



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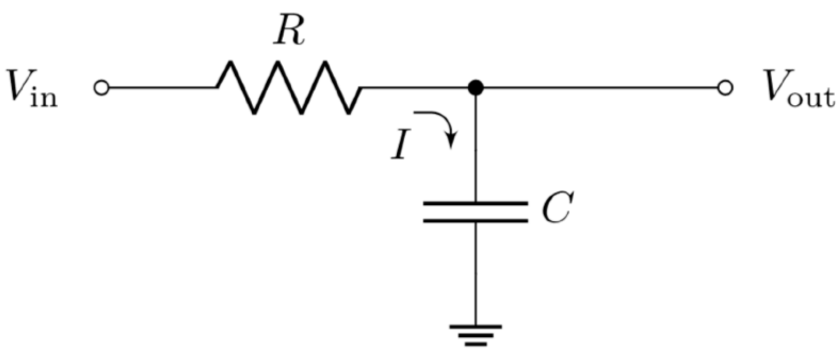

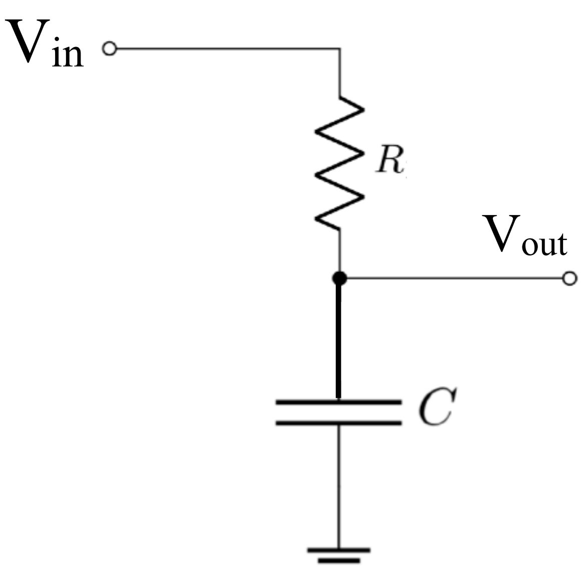

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$I = (V_{in}-V_{out})/R = C\, dV_{out}/dt$

Note that this assumes V_{out} is connected to a high input resistance.

Charging a capacitor

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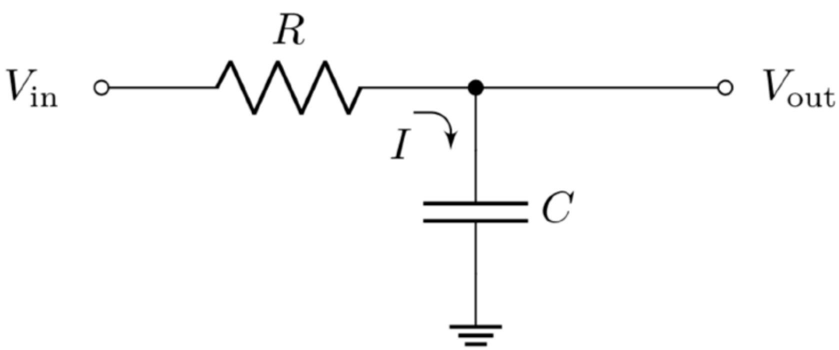

$I = (V_{in} - V_{out})/R = C\ dV_{out}/dt$

V_{out} is the time varying voltage, so just call it V(t), our variable in this DE.

$(V_{in} - V)/R = C\ dV/dt$

Suppose V_{in} is a step function at t=0.

Separate variables and we get $dt/RC = dV/(V_{in} - V) = -dV/(V - V_{in})$

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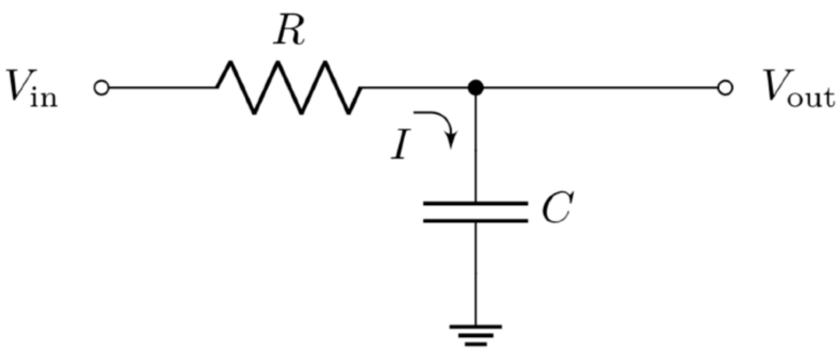

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Integrate both sides from t=0 to t=t and $V=V_0=V(t=0)$ to $V=V(t)$.

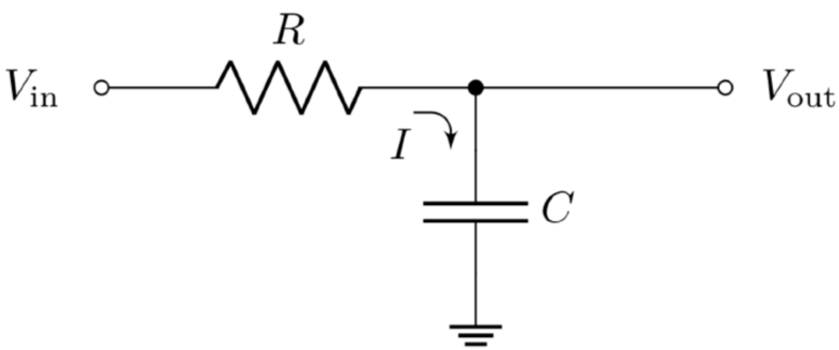
$$\int_0^t dt'/RC = -\int_{V_0}^{V} dV'/(V'-V_{in}) \quad \Rightarrow \quad t/RC = -\ln[(V-V_{in})/(V_0-V_{in})]$$

$$(V-V_{in}) = (V_0-V_{in})\, e^{-t/RC} \quad \Rightarrow \quad V = V_{in} + (V_0-V_{in})\, e^{-t/RC}$$

If $V_0=0$, we get $V(t) = V_{in}\,(1-e^{-t/RC})$

Discharging a capacitor

Suppose we charge a capacitor through a resistor with this circuit.



$I = (V_{in}\text{-}V_{out})/R = C\ dV_{out}/dt$

V_{out} is the time varying voltage, so just call it V(t), our variable in this DE.

$(V_{in}\text{-}V)/R = C\ dV/dt$

Suppose V_{in} is a step function at t=0.

Separate variables and we get $\quad dt/RC = dV/(V_{in}\text{-}V) = \text{-}dV/(V\text{-}V_{in})$

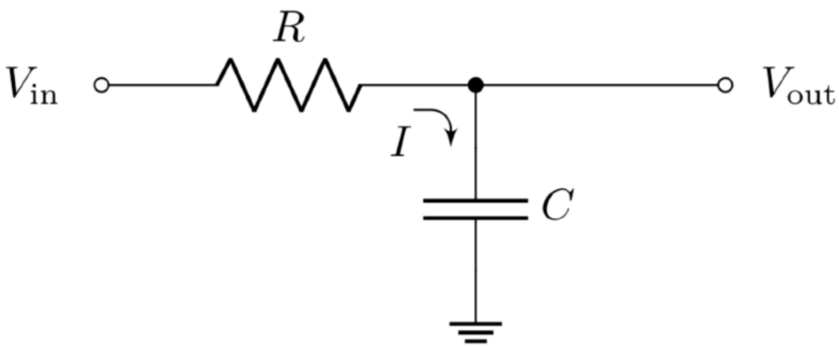
Integrate both sides from t=0 to t=t and $V=V_0=V(t=0)$ to $V=V(t)$.

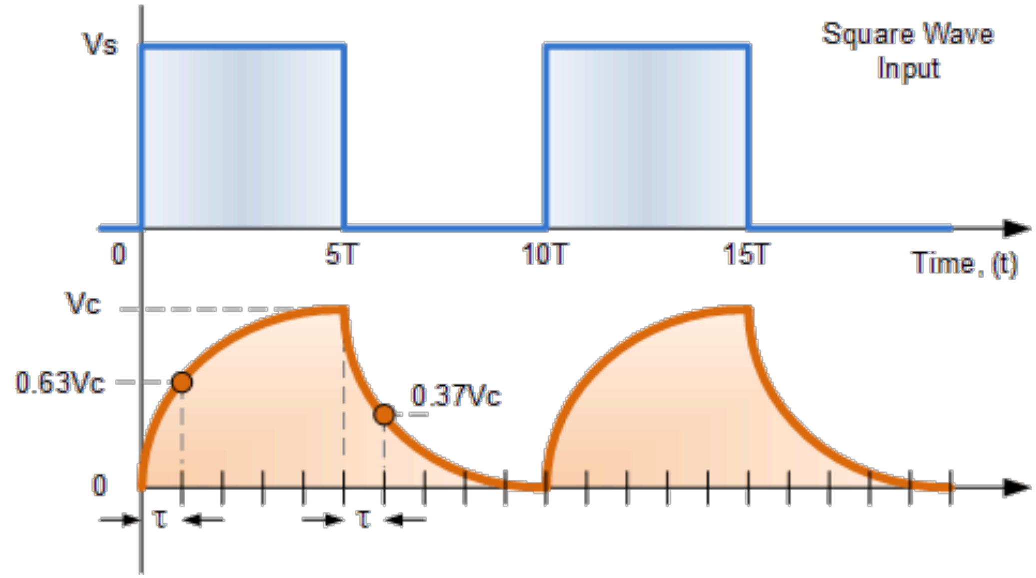
$$\int_0^t dt'/RC = -\int_{V_0}^{V} dV'/(V'\text{-}V_{in}) \quad \Rightarrow \quad t/RC = -\ln[(V\text{-}V_{in})/(V_0\text{-}V_{in})]$$

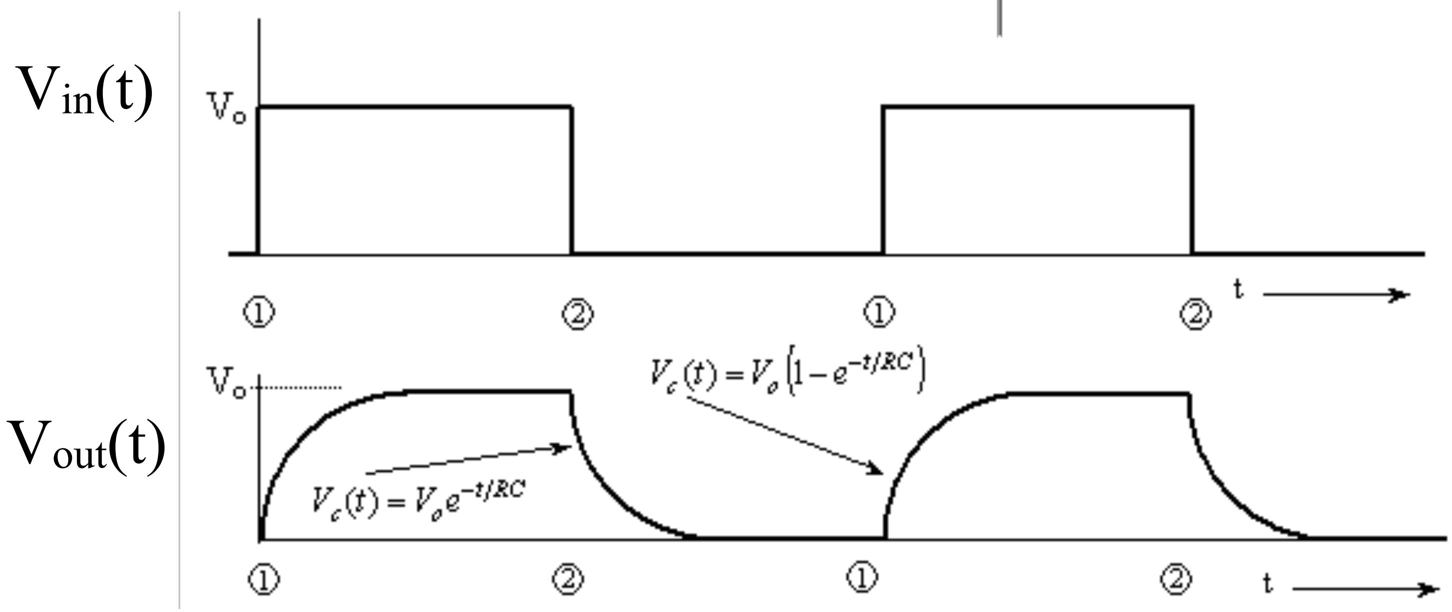
$$(V\text{-}V_{in}) = (V_0\text{-}V_{in})\ e^{-t/RC} \quad \Rightarrow \quad V = \cancelto{0}{V_{in}} + (V_0\text{-}\cancelto{0}{V_{in}})\ e^{-t/RC}$$

Now we get $V(t) = V_0\ e^{-t/RC}$

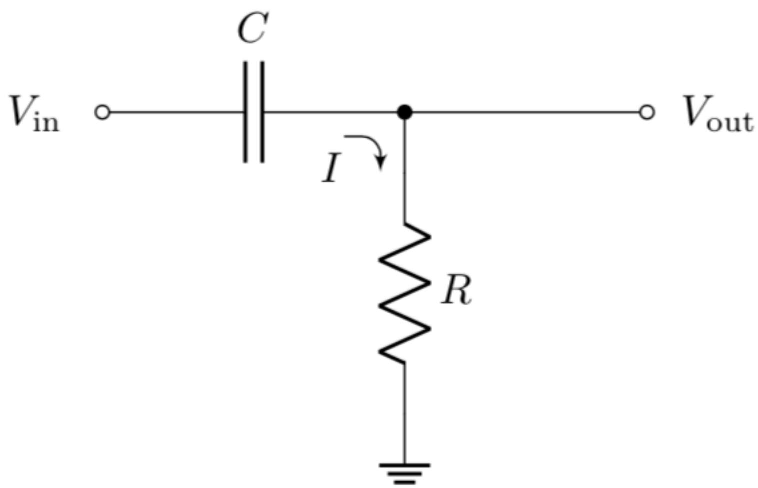
Square wave input to an RC circuit







Differentiator



We can again analyze this from

$q = CV \quad \Rightarrow \quad I = C\,dV/dt$

I *through* capacitor is related to the change in the voltage *across* the capacitor.

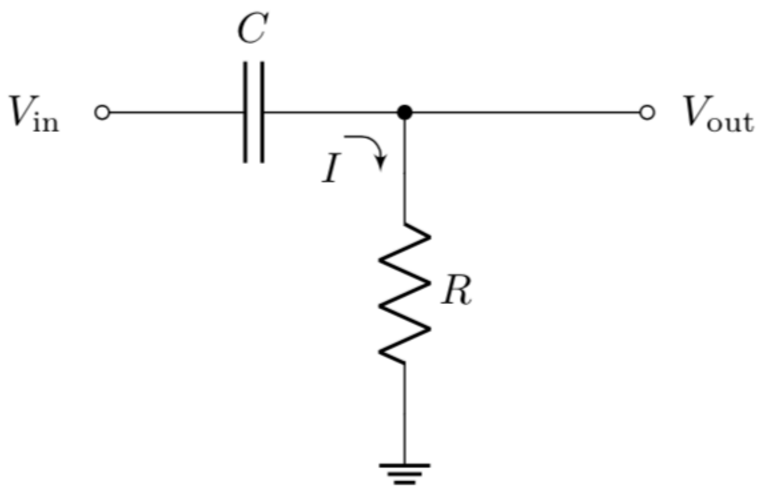
The voltage across the capacitor is V_{in}-V_{out}.

The current through the capacitor is whatever current is flowing through R from V_{out}, which is the voltage *dropped across* R:

$I = V_{out}/R = C\,d(V_{in}-V_{out})/dt$

Note that this assumes V_{out} is connected to a high input resistance.

Differentiator



$V_{out}/R = C\ d(V_{in}-V_{out})/dt$

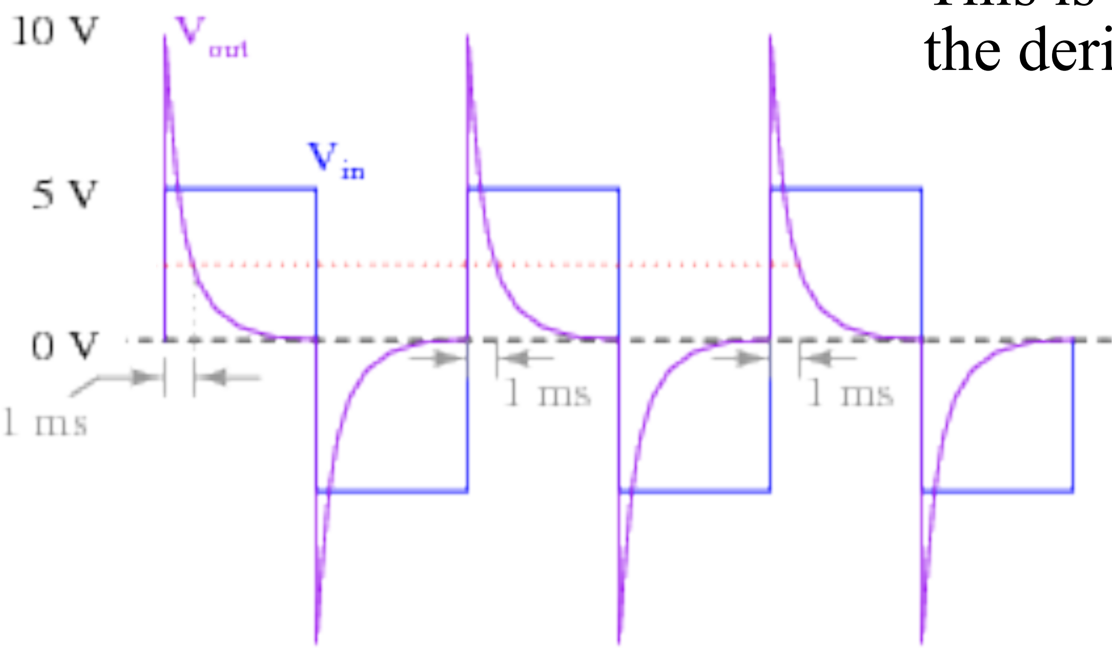
$V_{out}/RC = dV_{in}/dt - dV_{out}/dt$

If we assume that $dV_{out}/dt \ll dV_{in}/dt$ then we can simplify to

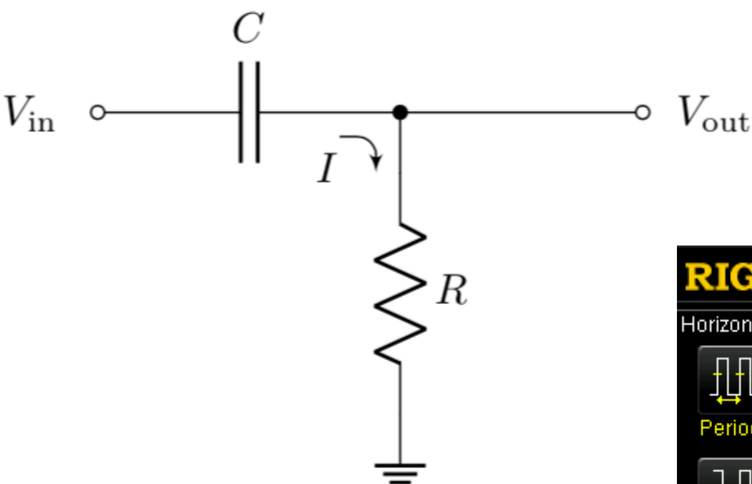
$V_{out}/RC = dV_{in}/dt$

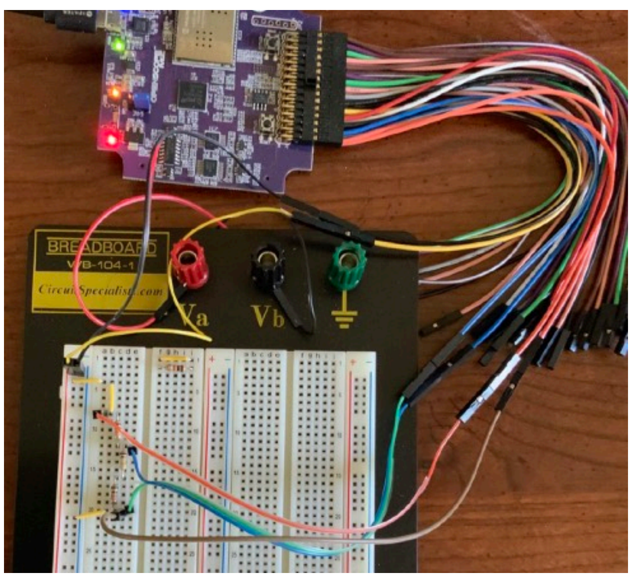
$V_{out}(t) = RC\ dV_{in}/dt$

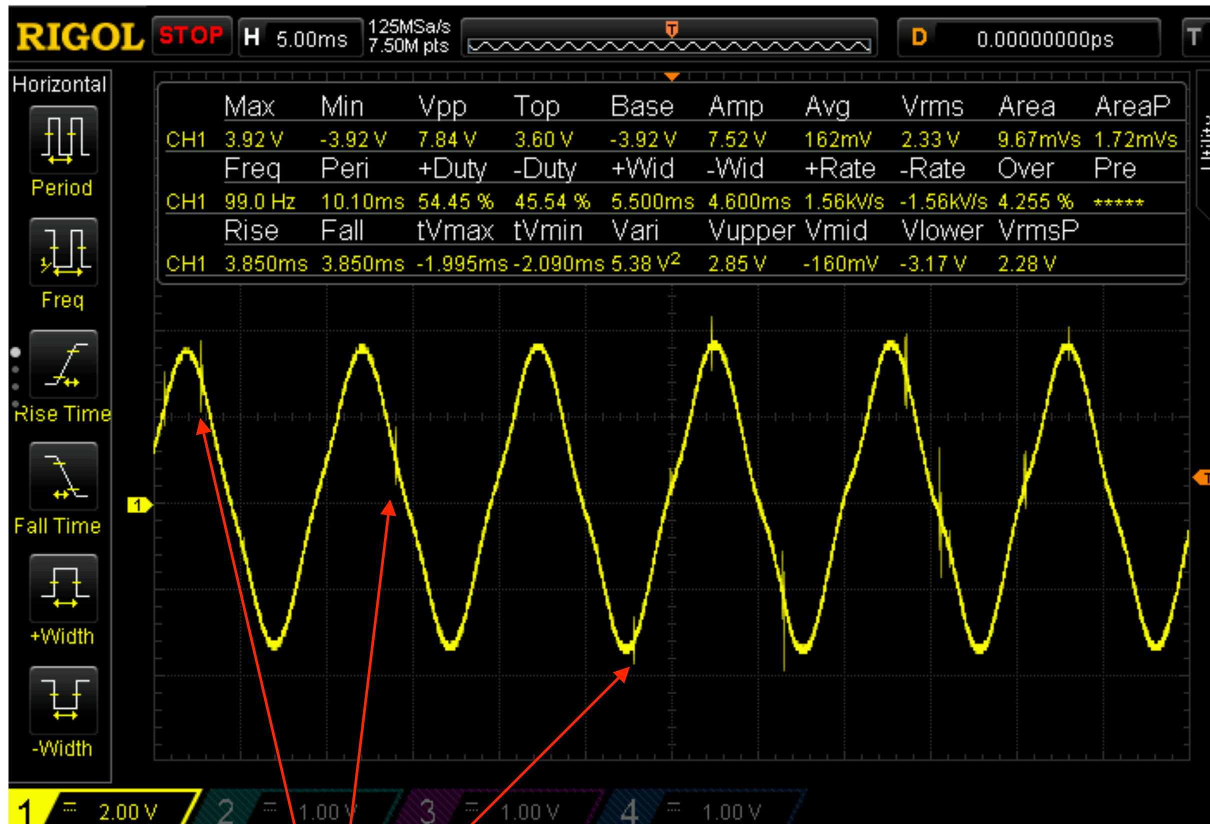
This is a differentiator; the output is the derivative of the input.



Differentiator pickup







Capacitive pickup from a square wave

Ohm's law for capacitors

We can find an IV relationship for a capacitor. Let's assume that V(t) is sinusoidal; we can then form any AC signal from a Fourier sum of these.

I'll actually use a cosine for reasons that will become clear later.

Suppose the voltage across the capacitor is

$V(t) = V_0 \cos \omega t$

Then the current through it is

$I(t) = C\, dV(t)/dt = -\omega C V_0 \sin \omega t$

So we have a relation between the magnitudes of V and I

$|V| = |I|\, (1/\omega C)$

This is similar to Ohm's law, and we can identify $1/\omega C$ as being like the "resistance of a capacitor". But it is not the full story because the *phase* of the current differs from the phase of the voltage.

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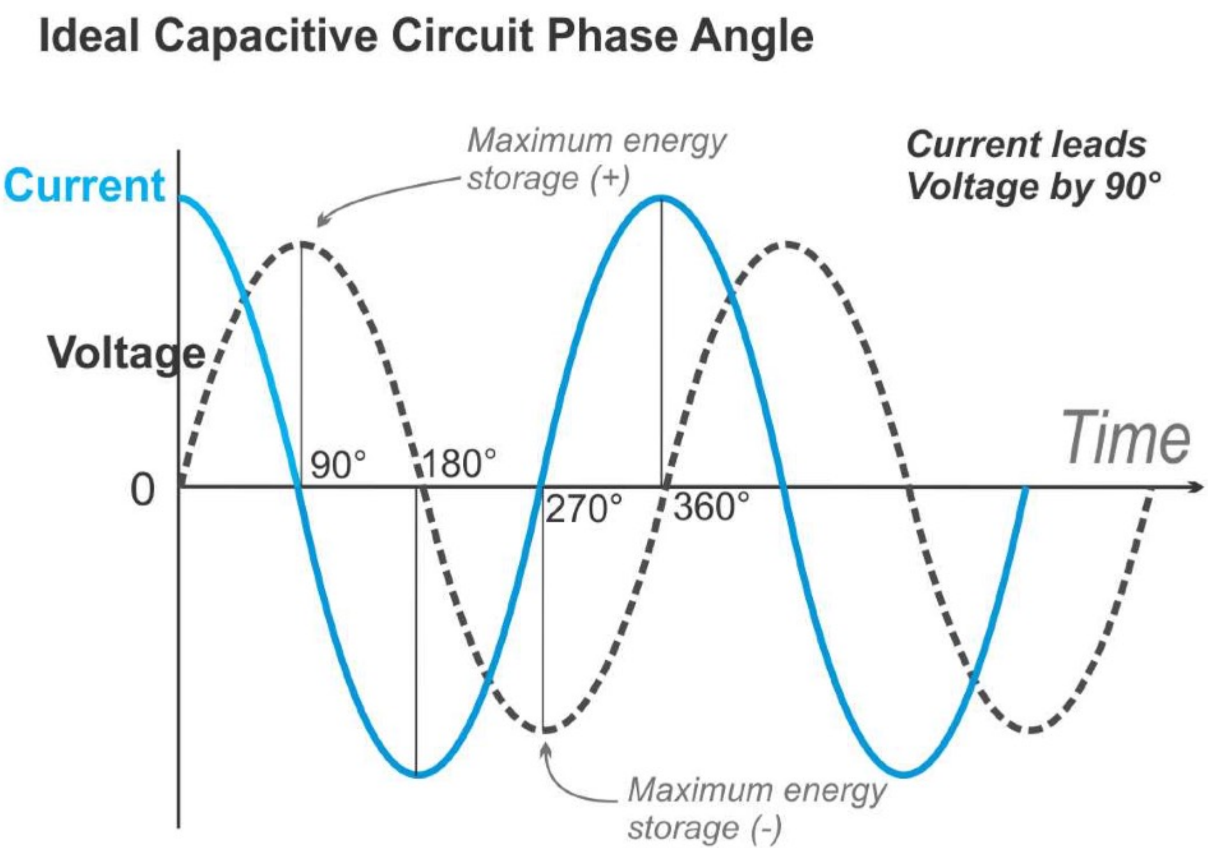

This diagram has V(t) as a sine not a cosine, but just shift time.

Ohm's law for capacitors

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Since the voltage "reacts to the current", and vice versa, this is called *reactance* rather than resistance.

The more general term is *impedance*, which encompasses both resistance and reactance. Impedance is the term to use for both henceforth.

So the impedance of a capacitor is $1/\omega C$.

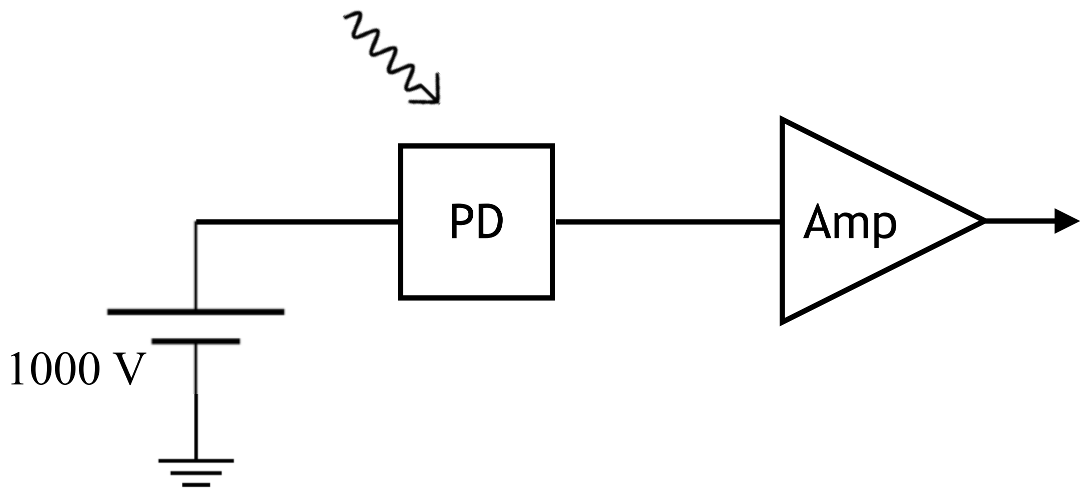
The minus sign indicates the phase information; though a rigorous treatment of that is more complex as we will see soon.

The $1/\omega C$ means that the impedance of a capacitor depends on the frequency of the signal.

Low impedance for high frequency AC and high impedance for DC.

DC blocking capacitors

The infinite resistance at DC is used to block DC from one stage of a circuit reaching another stage.



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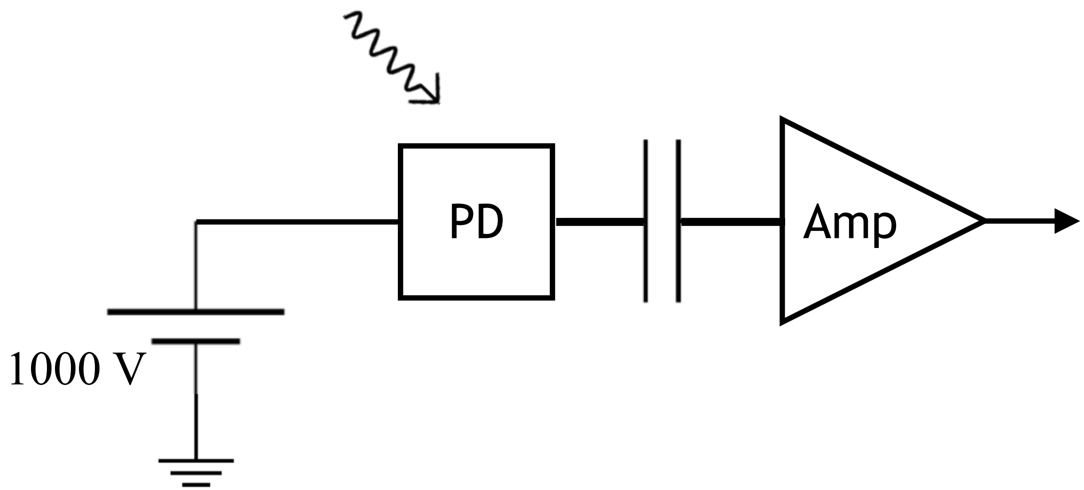


In addition to a capacitance value, you will find a voltage rating on capacitors indicating how much they can stand off.

Expressing AC signals in complex notation

It simplifies the handling of AC signals to treat them with complex notation. We can do this with Euler's formula.

$e^{i\theta} = \cos\theta + i \sin\theta$

So we can express $V(t) = V_0 \cos \omega t$ as $V(t) = V_0\, e^{i\omega t}$ well the real part is V(t).

We already use *i* for a small signal change in current, so in electronics we instead use $j^2 = -1$. (Textbook uses $j=-i$). So we'll represent the voltage as

$$\tilde{V}(t) = V_0\, e^{j\omega t}$$

The tilde reminds us that this is the complex representation. Now calculate the current from I = C dV/dt.

$$\tilde{I} = C\, j\omega V_0\, e^{j\omega t} = j\omega C \tilde{V}$$

So, we can write an Ohm's law like relation between V and I:

$$\tilde{V} = \tilde{I}\,(1/j\omega C) = \tilde{I}\,(-j/\omega C) \quad \Rightarrow \quad \tilde{V} = \tilde{I}\,\tilde{X}_C \quad \text{cf} \quad \tilde{V} = \tilde{I}\,\tilde{X}_R$$

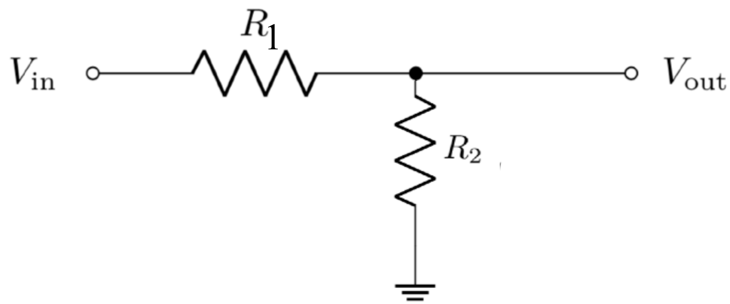
Where the impedance of the capacitor in complex representation is

$$\tilde{X}_C = -j/\omega C \quad \text{cf} \quad \tilde{X}_R = R$$

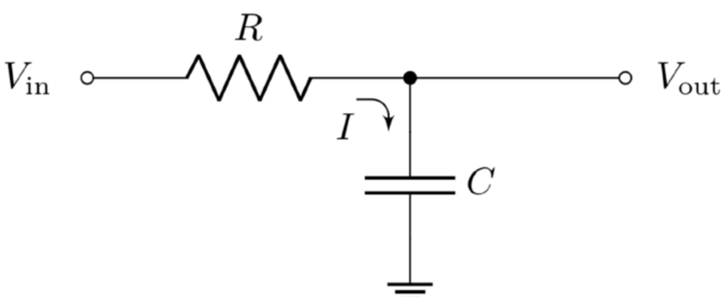
Expressing AC signals in complex notation

This will simplify handling a mix of capacitors and resistors.

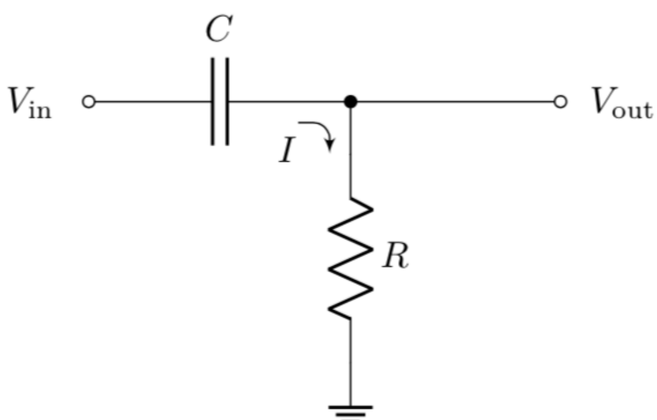
This makes the circuits below just complex voltage dividers.



$$V_{out} = IR_2 = V_{in}\frac{R_2}{R_1+R_2} = \widetilde{V}_{in}\frac{\widetilde{X}_2}{\widetilde{X}_1+\widetilde{X}_2}$$



$$\widetilde{V}_{out} = \widetilde{I}\,\widetilde{X}_C = \widetilde{V}_{in}\frac{\widetilde{X}_C}{R+\widetilde{X}_C}$$



$$\widetilde{V}_{out} = \widetilde{I}\,\widetilde{X}_R = \widetilde{V}_{in}\frac{R}{R+\widetilde{X}_C}$$

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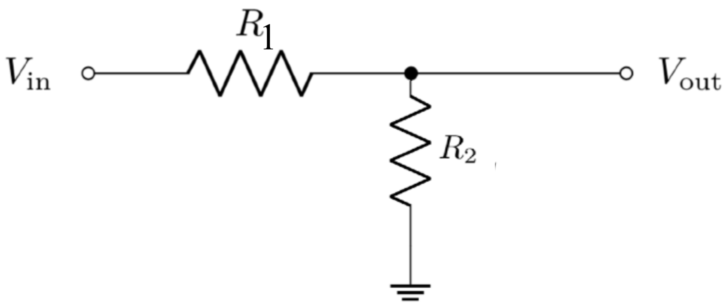

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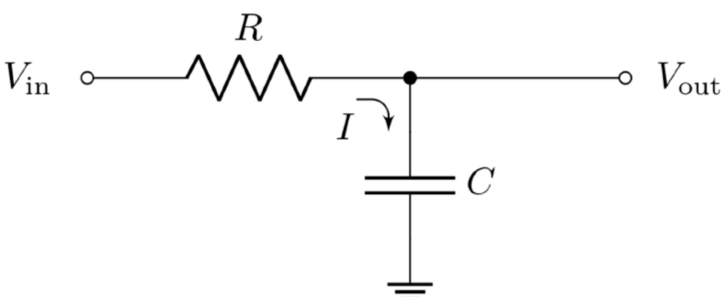

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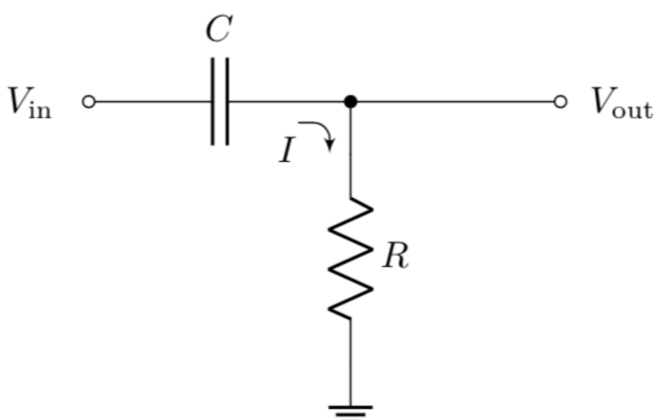
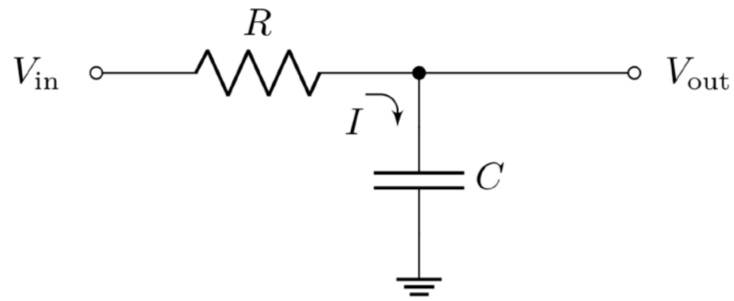


$$\widetilde{V}_{out} = \widetilde{I}\,\widetilde{X}_R = \widetilde{V}_{in}\frac{R}{R+\widetilde{X}_C}$$

We can use this to calculate the output voltage in terms of input voltage, V_{out}/V_{in}.

Calculating V_{out}/V_{in} with complex impedance

$$\tilde{V}_{\text{out}} = \tilde{I}\tilde{X}_C = \frac{\tilde{V}_{\text{in}}}{R+\tilde{X}_C}\tilde{X}_C = \tilde{V}_{\text{in}}\frac{\tilde{X}_C}{R+\tilde{X}_C}$$



Let's leave the voltages as arbitrary, but put in the explicit form for $\tilde{X}_C$.

$$\tilde{V}_{\text{out}} = \tilde{V}_{\text{in}}\left[\frac{-j/\omega C}{R-j/\omega C}\right]$$

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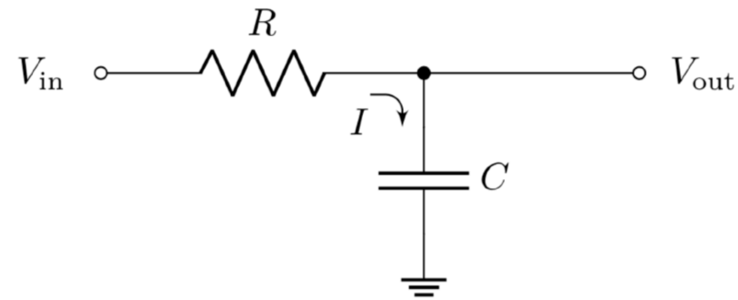

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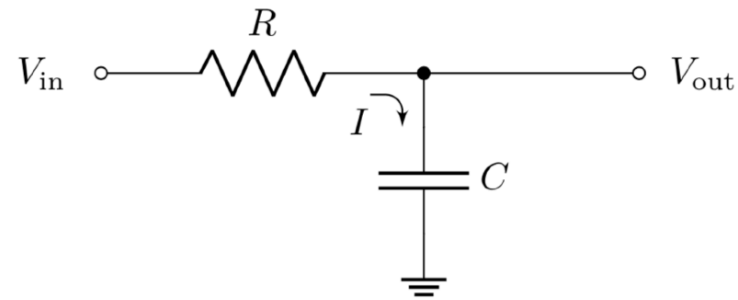

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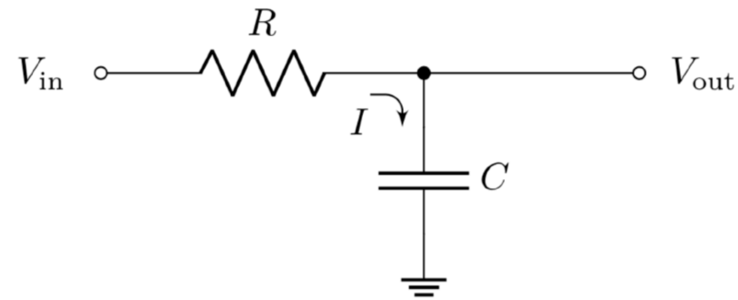
$$\tilde{V}_{\text{out}} = \tilde{V}_{\text{in}}\frac{1/\omega^2 C^2 - jR/\omega C}{R^2+1/\omega^2 C^2} * \left[\frac{\omega^2 C^2}{\omega^2 C^2}\right] = \tilde{V}_{\text{in}}\frac{1-j\omega RC}{1+\omega^2 R^2 C^2}$$

Calculating V_{out}/V_{in} with complex impedance

$$\tilde{V}_{\text{out}} = \tilde{I}\tilde{X}_C = \frac{\tilde{V}_{\text{in}}}{R+\tilde{X}_C}\tilde{X}_C = \tilde{V}_{\text{in}}\frac{\tilde{X}_C}{R+\tilde{X}_C}$$



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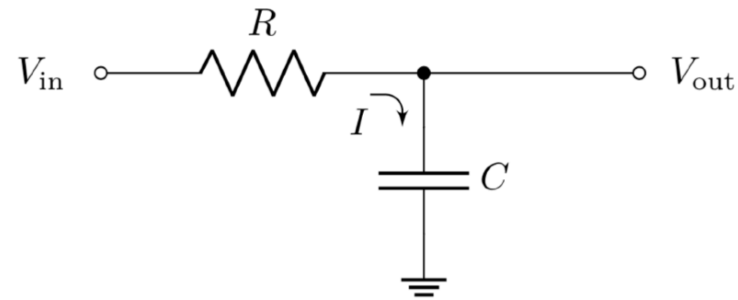
$$|\tilde{V}_{\text{out}}| = |\tilde{V}_{\text{in}}|\left|\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right|$$

$$\frac{|\tilde{V}_{\text{out}}|}{|\tilde{V}_{\text{in}}|} = \left|\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right|$$

Calculating V_{out}/V_{in} with complex impedance

$$\tilde{V}_{\rm out} = \tilde{I}\tilde{X}_C = \frac{\tilde{V}_{\rm in}}{R+\tilde{X}_C}\tilde{X}_C = \tilde{V}_{\rm in}\frac{\tilde{X}_C}{R+\tilde{X}_C}$$



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$$|\tilde{V}_{\rm out}| = |\tilde{V}_{\rm in}|\left|\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right|$$

$$\frac{|\tilde{V}_{\rm out}|}{|\tilde{V}_{\rm in}|} = \left|\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right| = \sqrt{\left[\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right]\left[\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right]^*} = \sqrt{\left[\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right]\left[\frac{1+j\omega RC}{1+\omega^2R^2C^2}\right]}$$

$$\frac{|\tilde{V}_{\rm out}|}{|\tilde{V}_{\rm in}|} = \sqrt{\frac{1+\omega^2R^2C^2}{(1+\omega^2R^2C^2)^2}} = \frac{\sqrt{1+\omega^2R^2C^2}}{1+\omega^2R^2C^2} = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

This is the response function for the circuit,
and it is frequency dependent through $\omega = 2\pi f$.