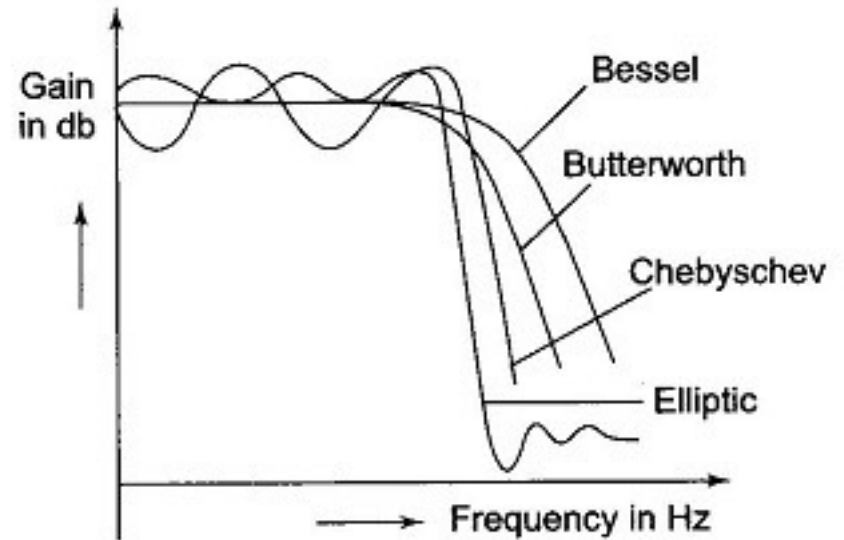
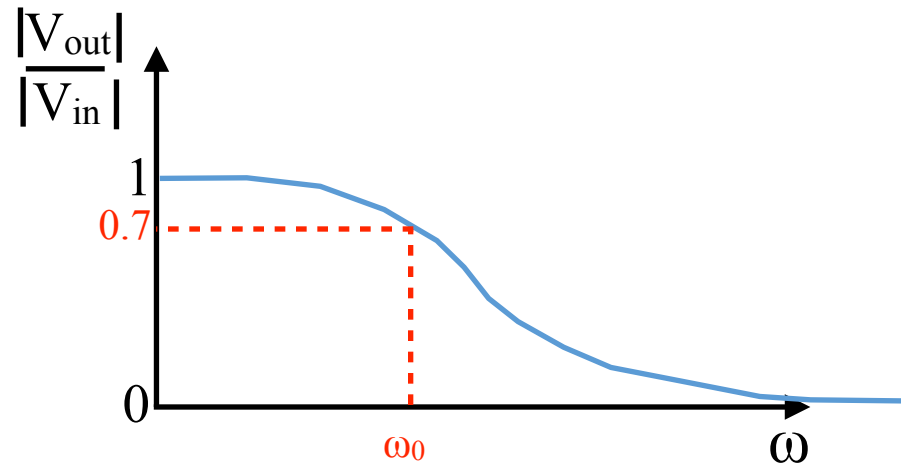


PHYS127AL Lecture 16

David Stuart, UC Santa Barbara

Active filters



Review: Frequency response function for a high-pass filter

$$\tilde{V}_{\text{out}} = \tilde{I} \tilde{X}_R = \tilde{I} R = \frac{\tilde{V}_{\text{in}}}{R + \tilde{X}_C} R = \tilde{V}_{\text{in}} \frac{R}{R + \tilde{X}_C}$$

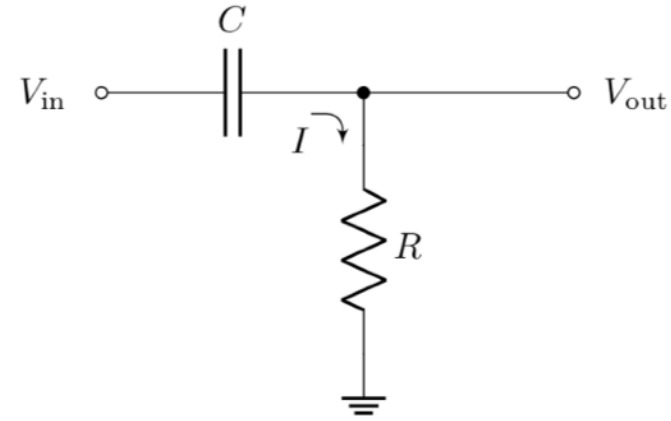
$$\tilde{V}_{\text{out}} = \tilde{V}_{\text{in}} \left[\frac{R}{R - j/\omega C} \right] = \tilde{V}_{\text{in}} \left[\frac{R}{R - j/\omega C} \right] \left[\frac{R + j/\omega C}{R + j/\omega C} \right]$$

$$\tilde{V}_{\text{out}} = \tilde{V}_{\text{in}} \frac{R^2 + jR/\omega C}{R^2 + 1/\omega^2 C^2} = \tilde{V}_{\text{in}} \frac{1 + j/\omega RC}{1 + 1/\omega^2 R^2 C^2}$$

$$|\tilde{V}_{\text{out}}| = |\tilde{V}_{\text{in}}| \left| \frac{1 + j\omega RC}{1 + \omega^2 R^2 C^2} \right|$$

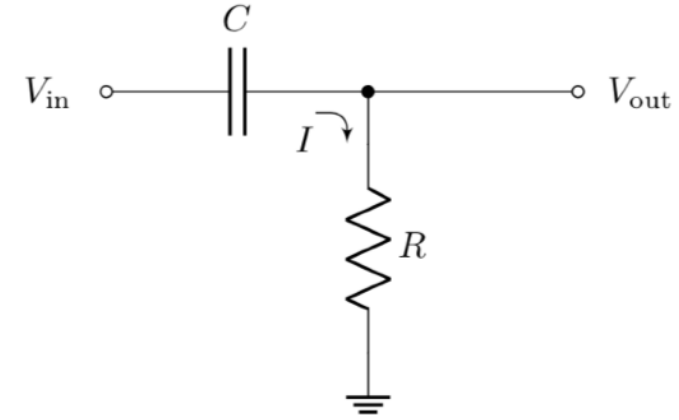
$$\frac{|\tilde{V}_{\text{out}}|}{|\tilde{V}_{\text{in}}|} = \left| \frac{1 + j/\omega RC}{1 + 1/\omega^2 R^2 C^2} \right| = \sqrt{\left[\frac{1 + j/\omega RC}{1 + 1/\omega^2 R^2 C^2} \right] \left[\frac{1 - j/\omega RC}{1 + 1/\omega^2 R^2 C^2} \right]}$$

$$\frac{|\tilde{V}_{\text{out}}|}{|\tilde{V}_{\text{in}}|} = \sqrt{\frac{1 + 1/\omega^2 R^2 C^2}{(1 + 1/\omega^2 R^2 C^2)^2}} = \frac{\sqrt{1 + 1/\omega^2 R^2 C^2}}{1 + 1/\omega^2 R^2 C^2} = \frac{1}{\sqrt{1 + 1/\omega^2 R^2 C^2}} = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$

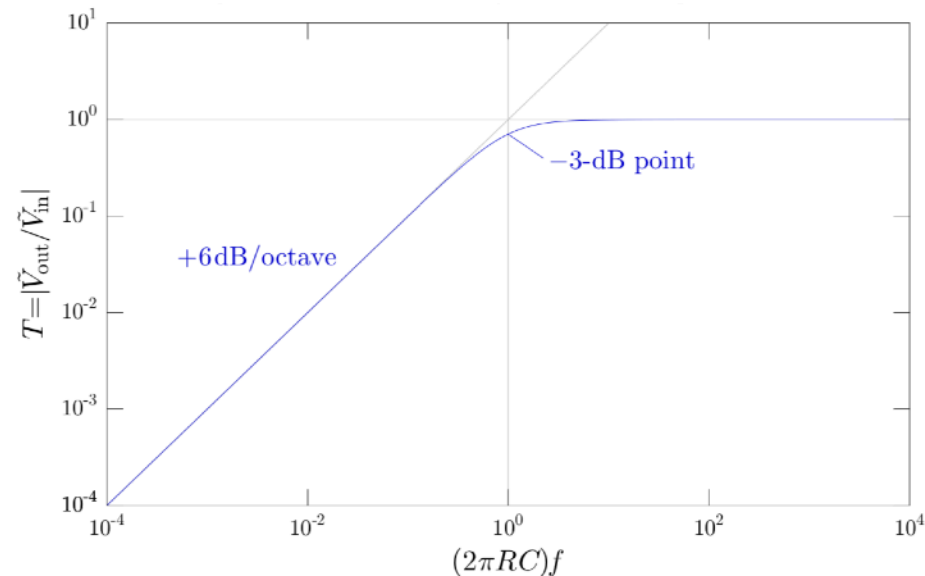
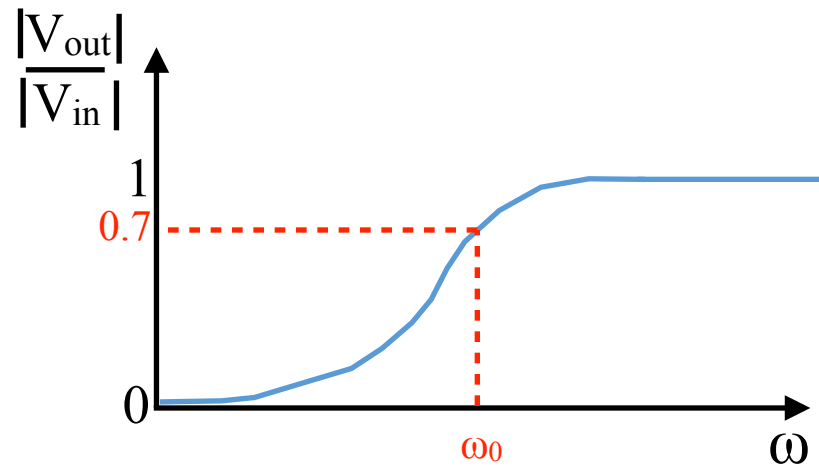


Review: Frequency response function for a high-pass filter

$$\frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \frac{1}{\sqrt{1+1/\omega^2 R^2 C^2}} = \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}$$



$V_{out} \rightarrow 0$ as $\omega \rightarrow 0$ and $V_{out} \rightarrow V_{in}$ as $\omega \rightarrow \infty$.



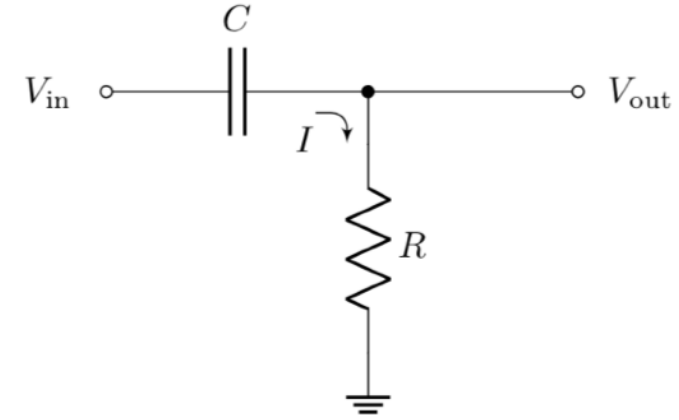
Can characterize the frequency scale with

$$\omega^2 = 1/R^2 C^2 \Rightarrow \omega = 1/RC$$

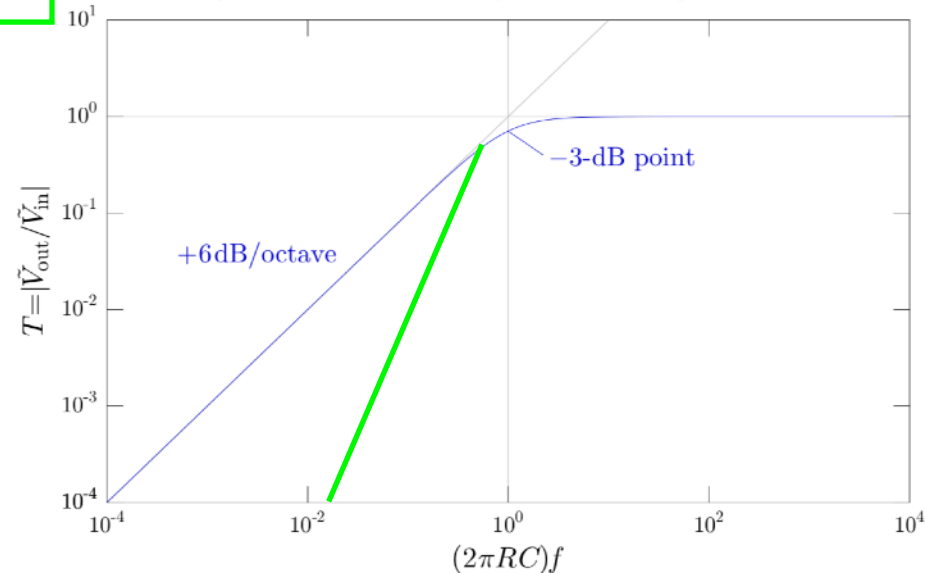
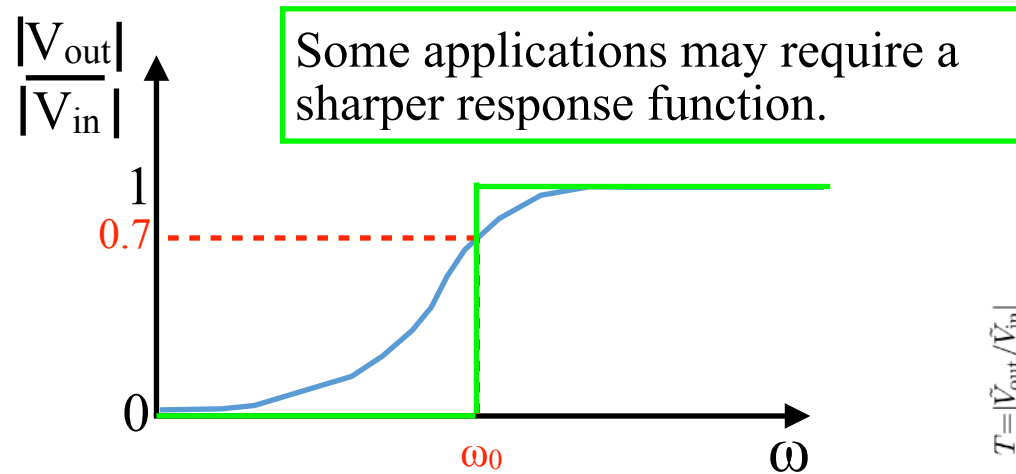
$$\frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \frac{1}{\sqrt{2}} = -3 \text{ dB}$$

Review: Frequency response function for a high-pass filter

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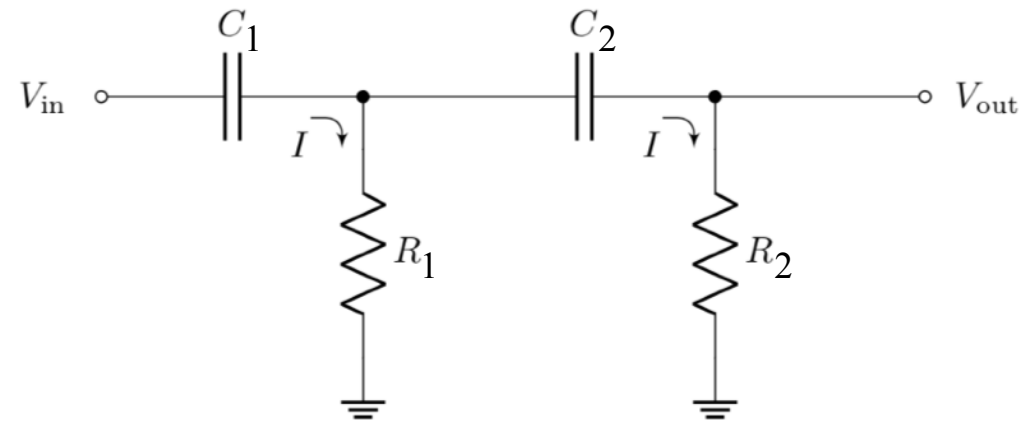


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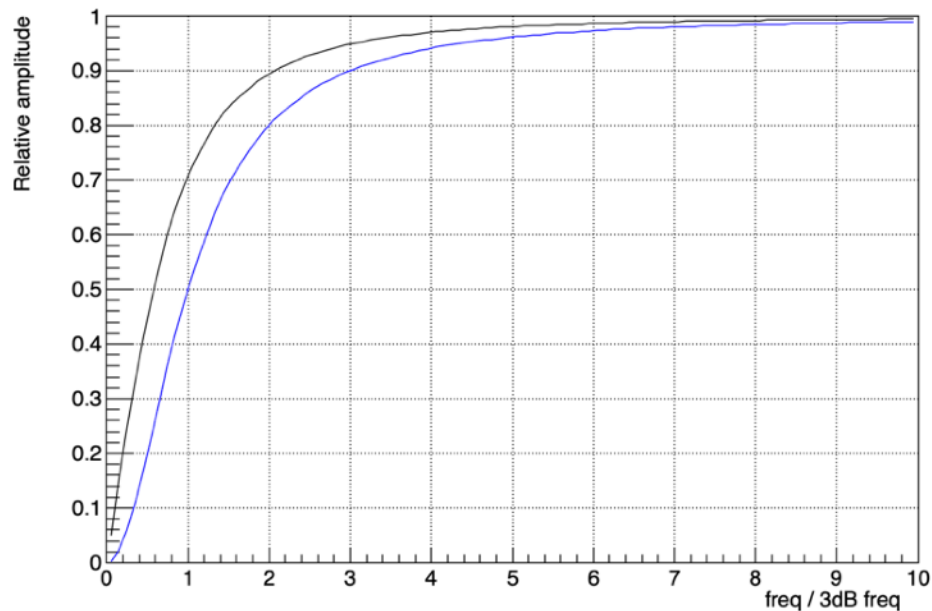
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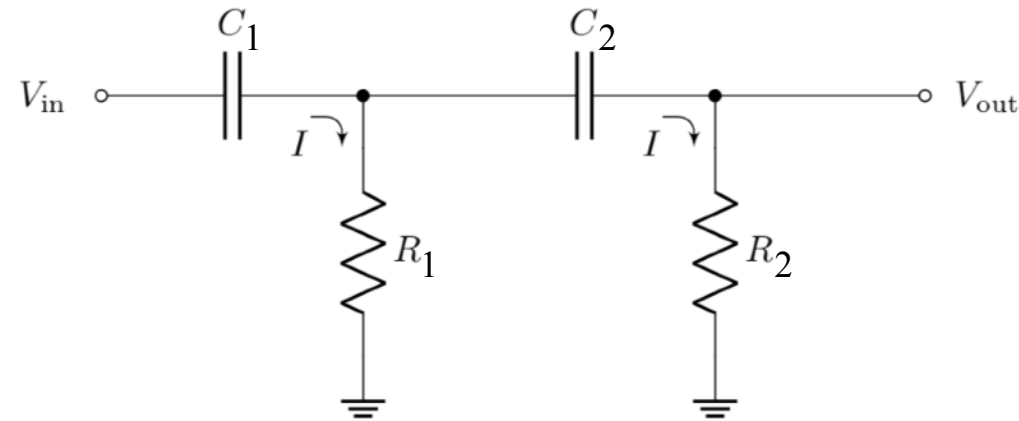
Sharpening the response function



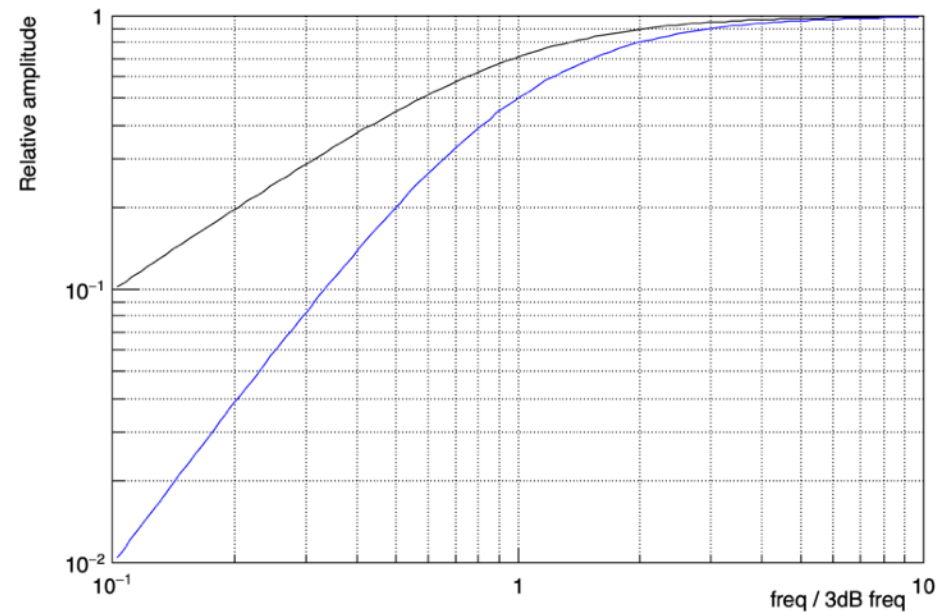
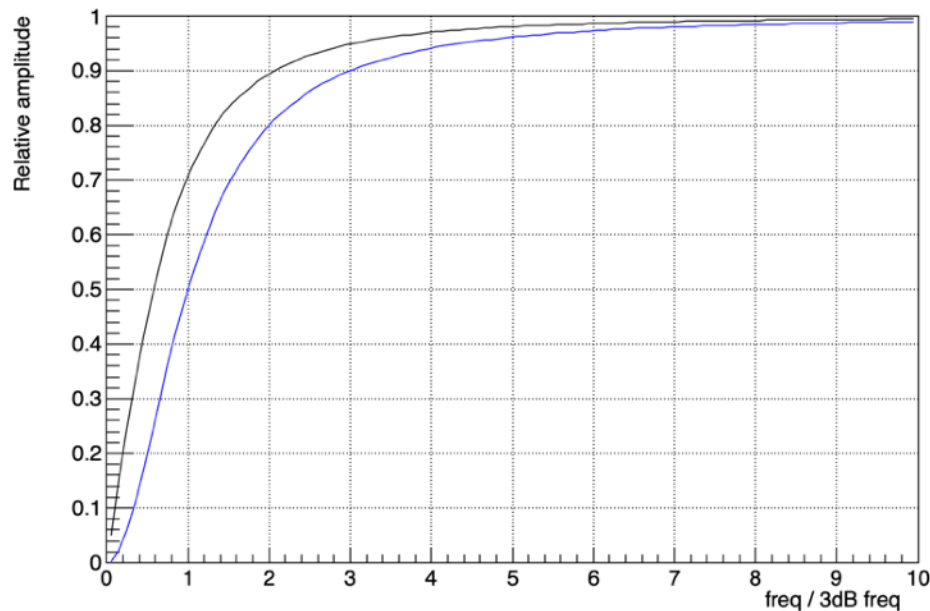
This improved the low-end frequency response, but we lose more at the high end.



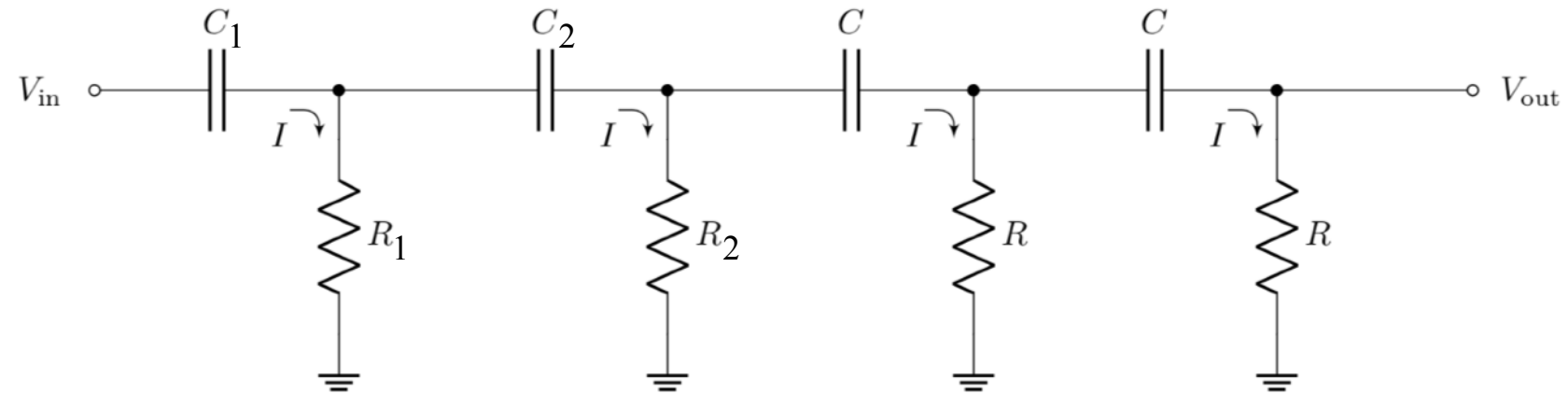
Sharpening the response function



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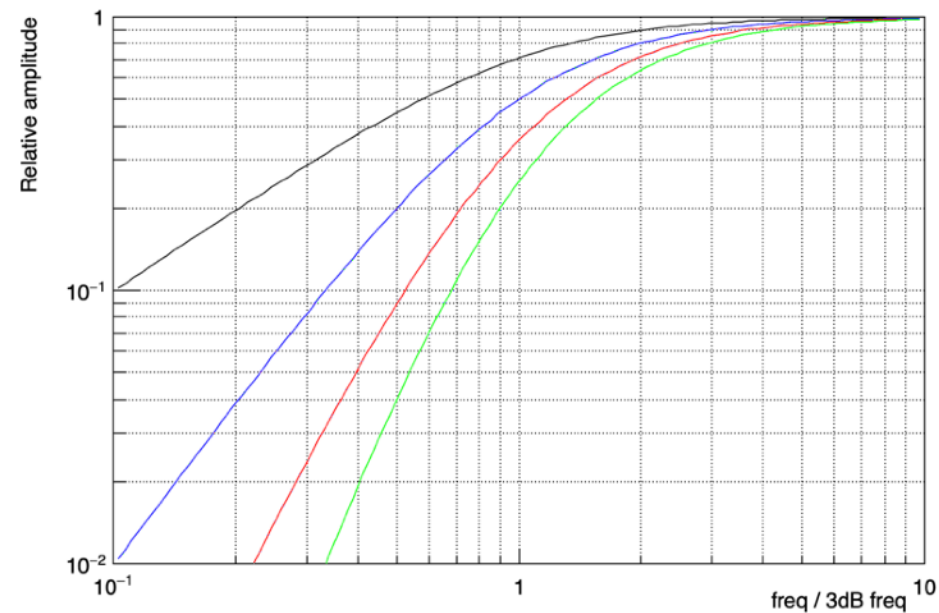
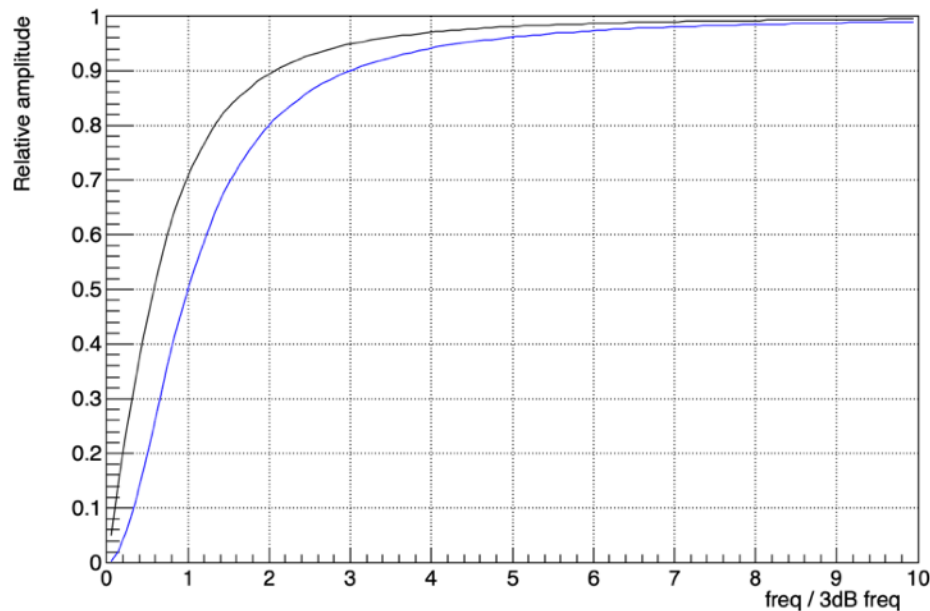


Sharpening the response function

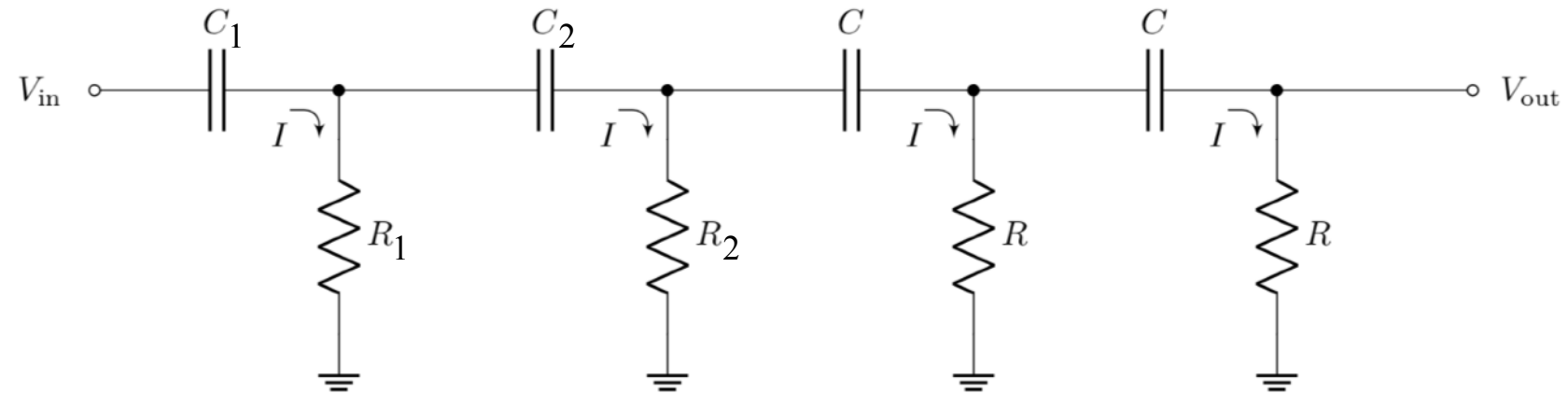


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We could add more stages to further suppress the low end.



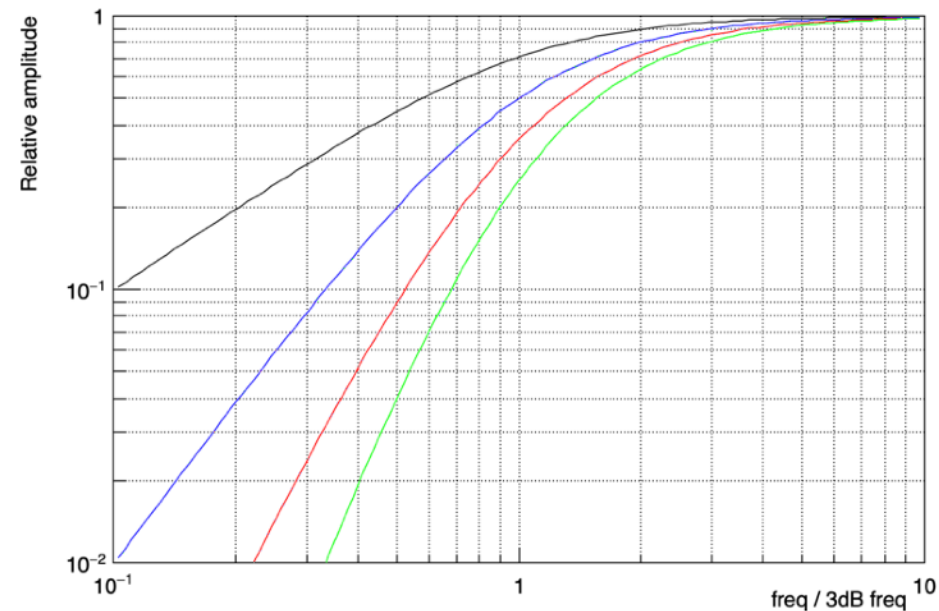
Sharpening the response function



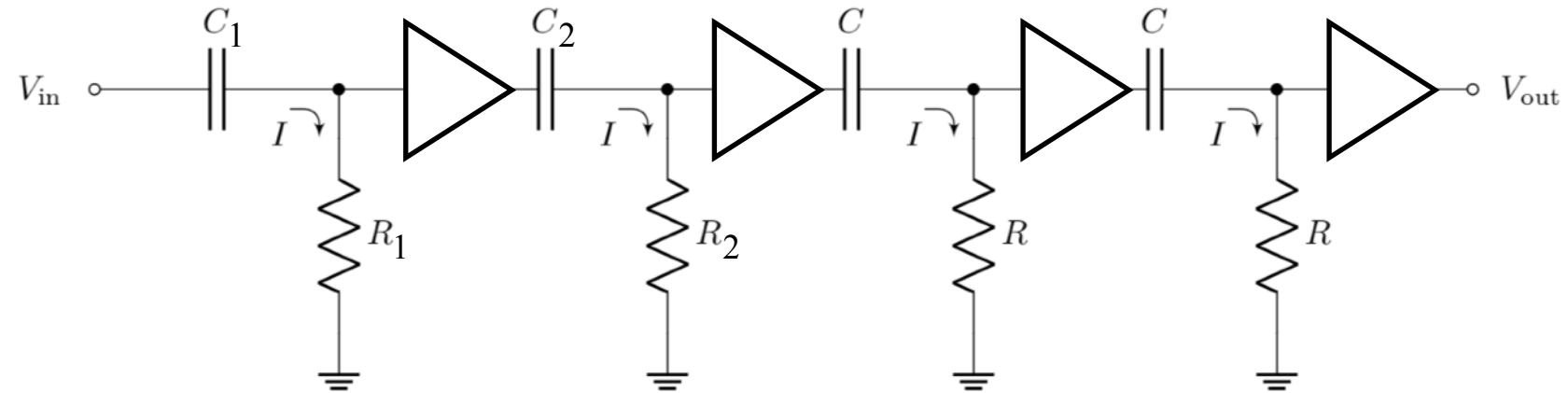
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Two problems: lose the high end and need $(\times 10)^4$ impedance increases.



Sharpening the response function

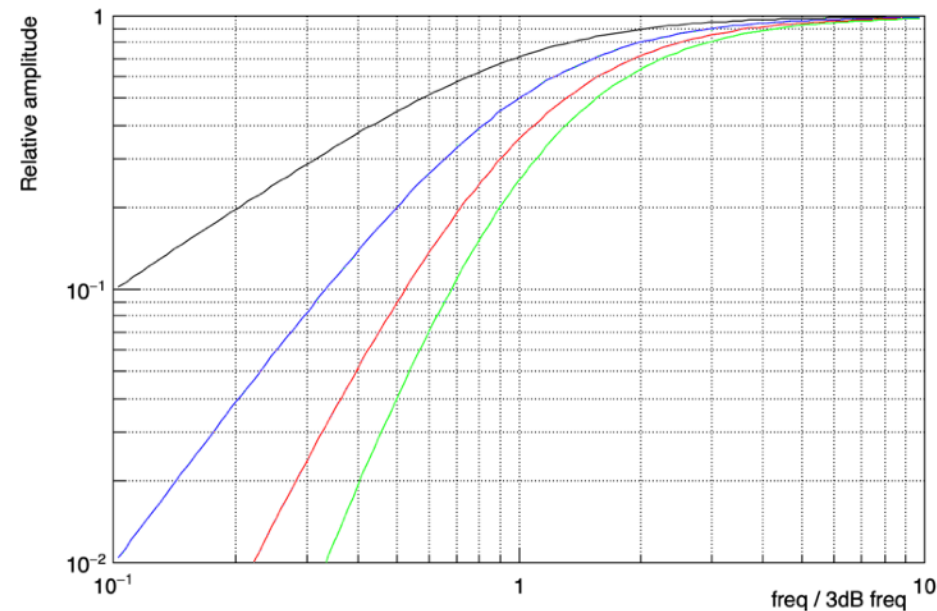


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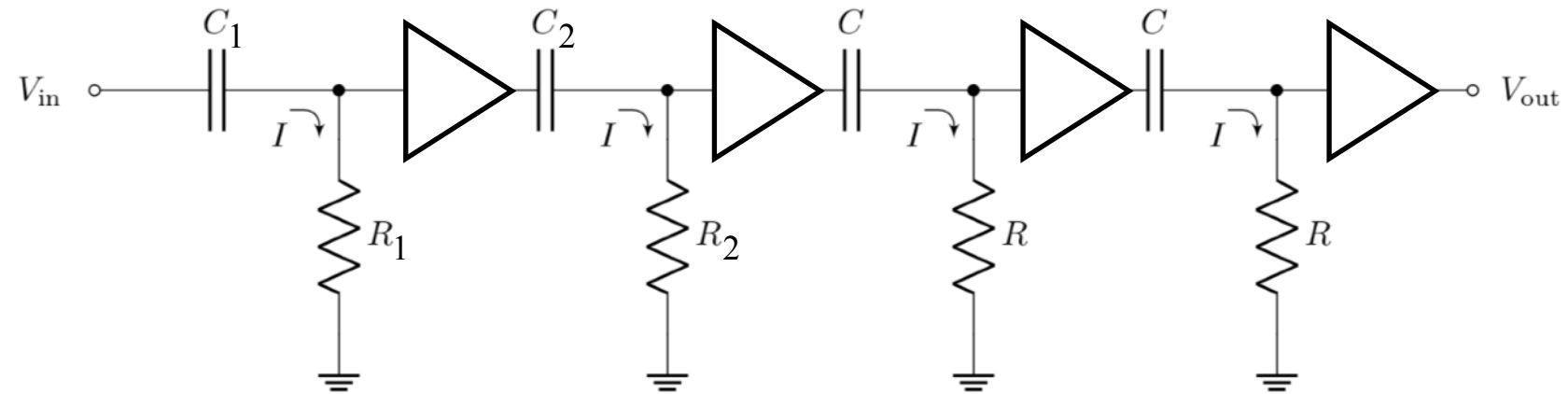
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Fix the impedance with op-amp buffers.



Sharpening the response function



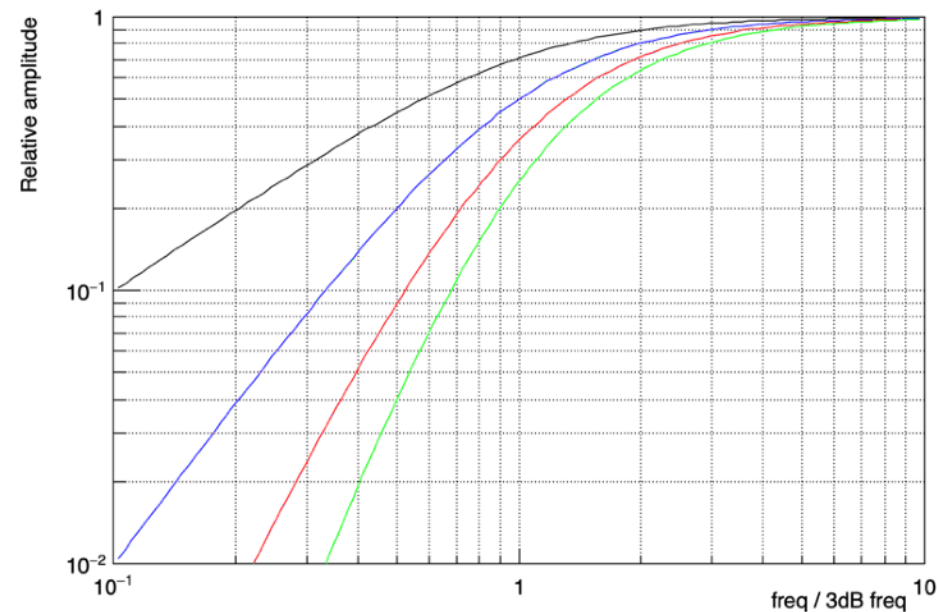
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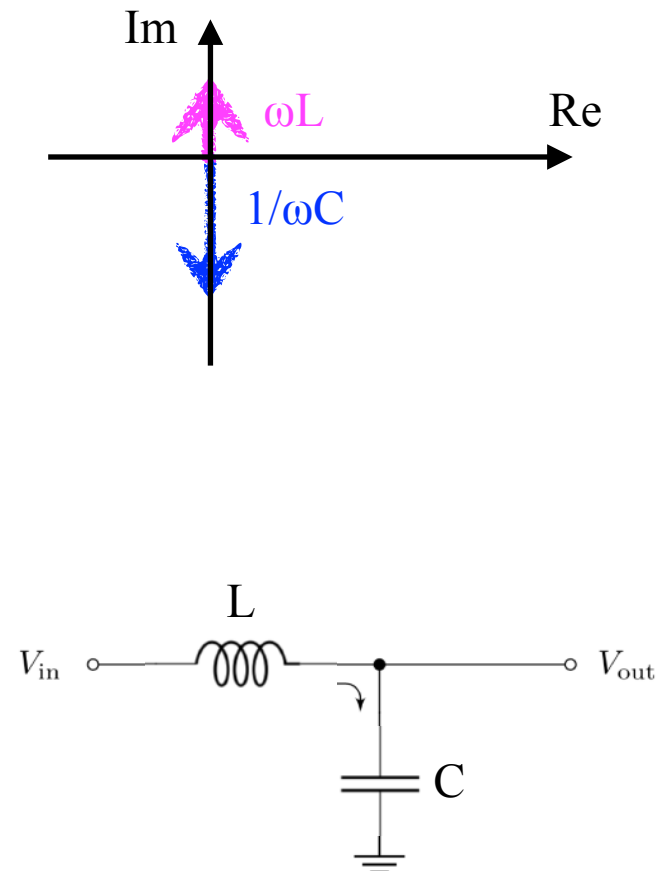
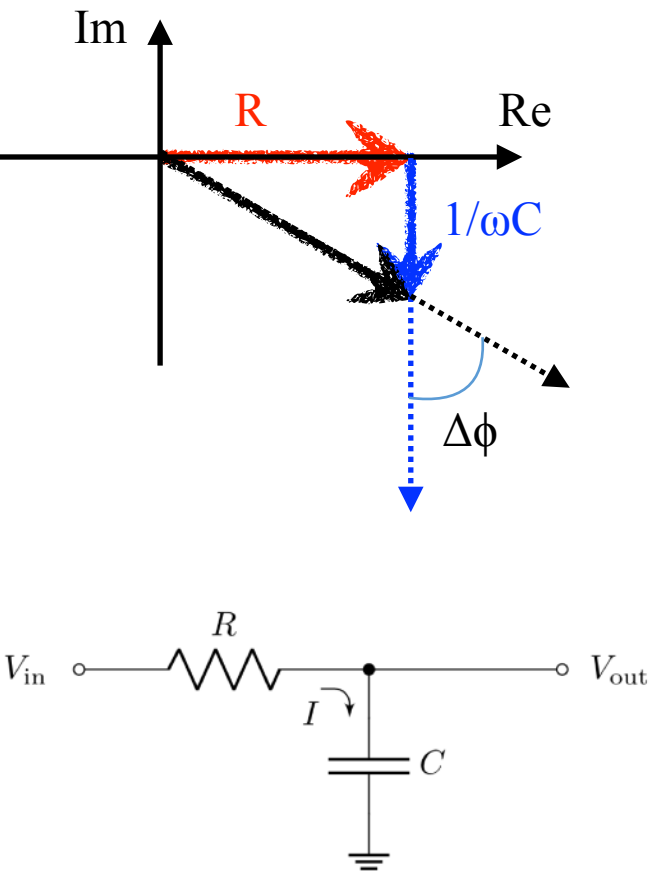
We could regain the high end by putting gain into the op-amp, and ideally make the gain be frequency dependent.



Sharpening the response function

Another problem is that each stage introduces a phase shift that could build up.

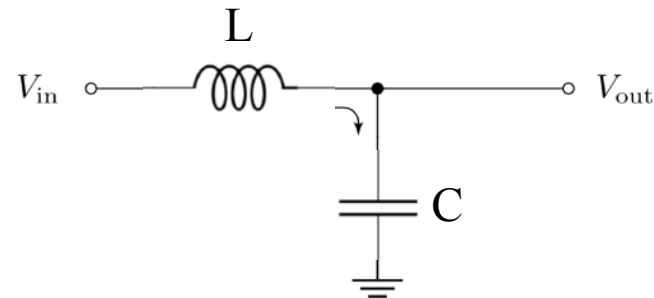
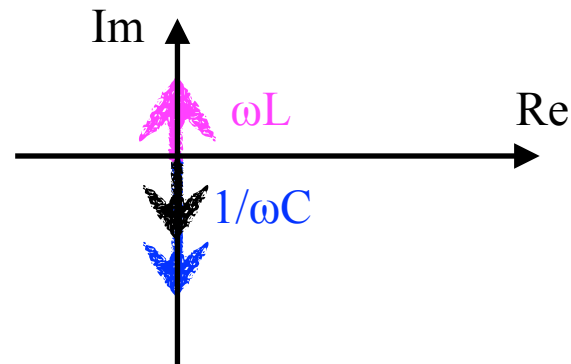
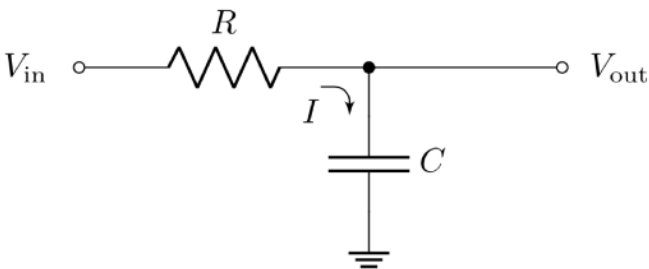
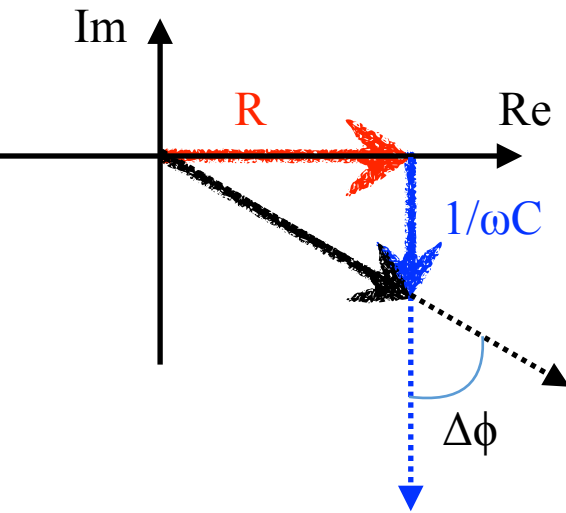
We could avoid the phase shift with an LC filter instead of RC.



Sharpening the response function

Another problem is that each stage introduces a phase shift that could build up.

We could avoid the phase shift with an LC filter instead of RC.



Shrinking an inductor

The problem with inductors is that they are physically huge.

Resistors and capacitors can be made small ($\sim\mu\text{m}$) with photolithography.

It would be nice to emulate an inductance with tiny components (like R, C, and op-amps)

$X_L = j\omega L$ while $X_C = -j/\omega C$.

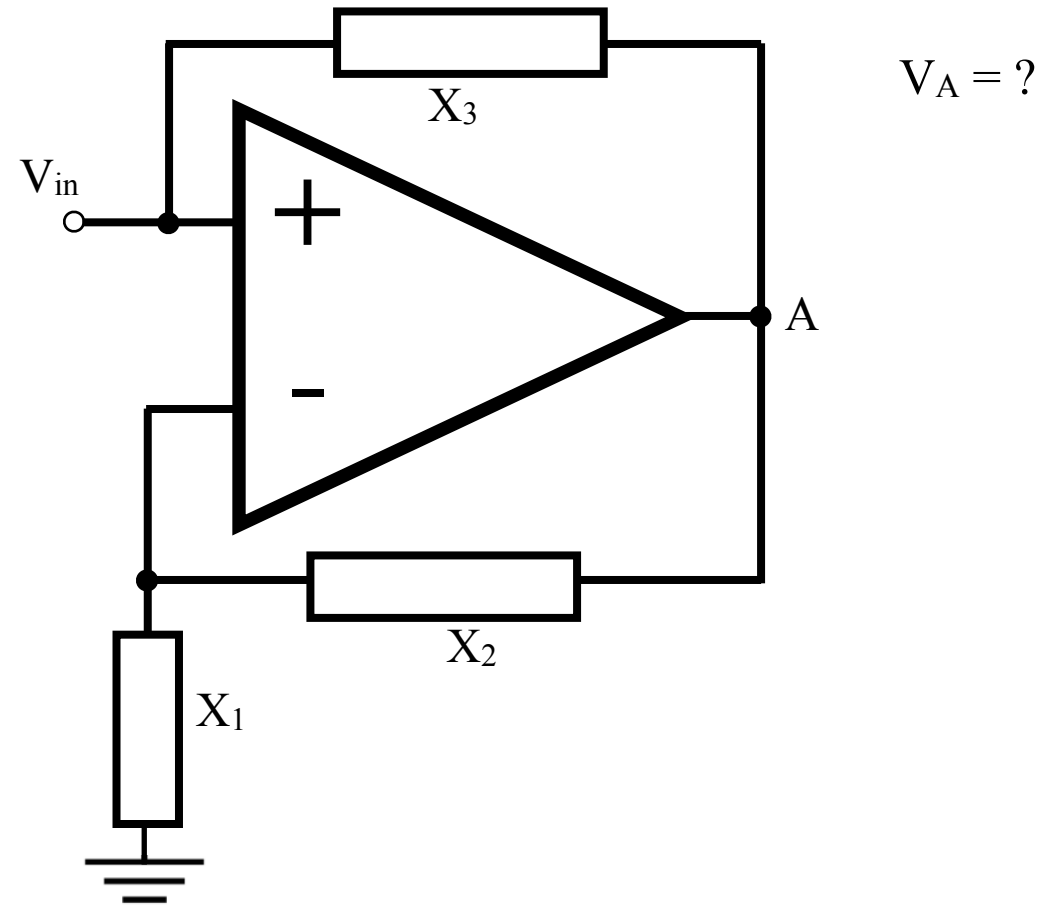
To convert a capacitance to an inductance we need to *invert* the frequency dependence and *negate* the impedance. (Negative impedance means that a higher voltage reduces the current.)

We can do that with a "negative impedance converter".

Negative impedance converter

The problem with inductors is that they are physically huge.

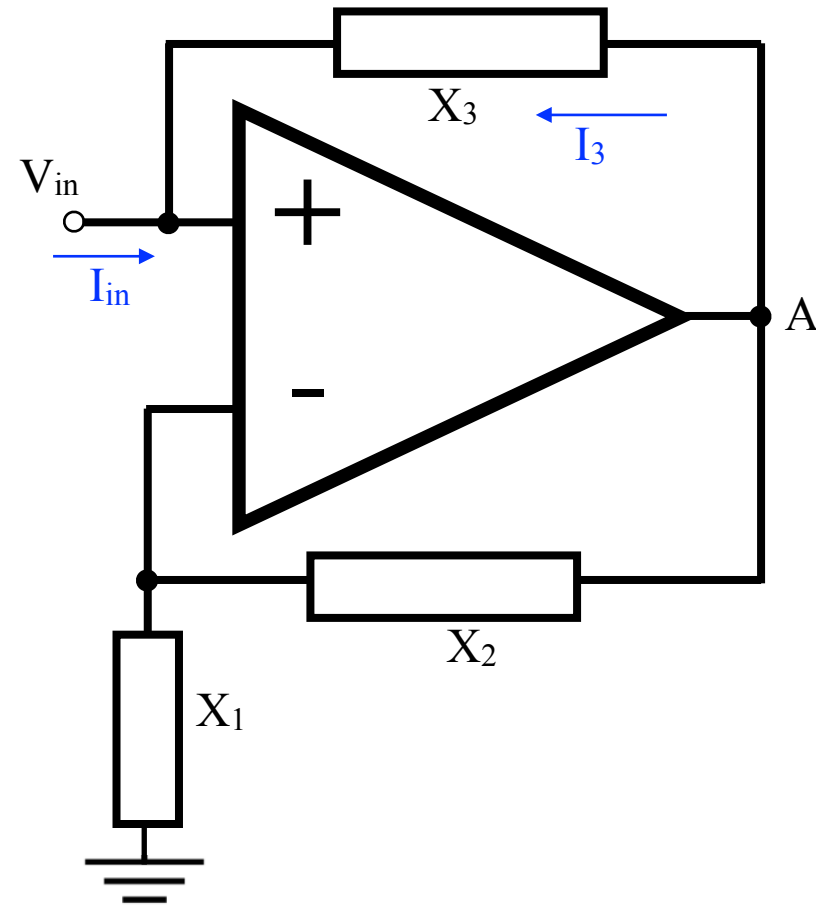
A negative impedance converter helps provide the inductor emulation.



Negative impedance converter

The problem with inductors is that they are physically huge.

A negative impedance converter helps provide the inductor emulation.



$$V_A = V_{in} (1 + X_2/X_1)$$

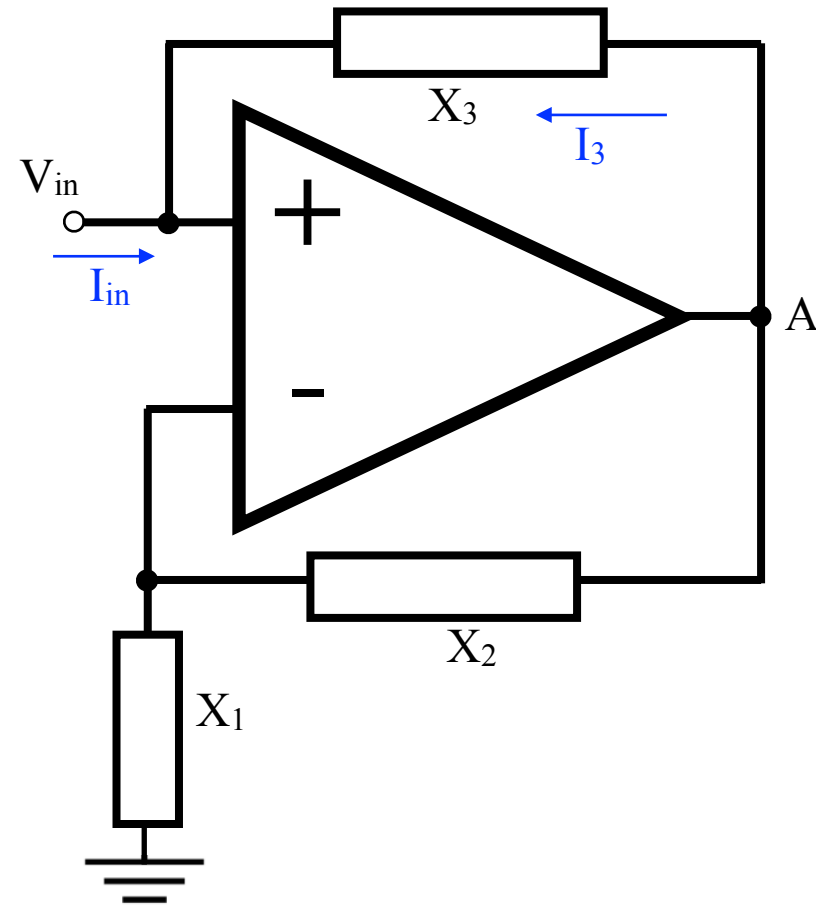
$$I_3 = (V_A - V_{in})/X_3 = (V_{in} + V_{in} X_2/X_1 - V_{in})/X_3 \\ = V_{in} X_2/(X_1 X_3)$$

I_3 flows *from A into* the input, so $I_{in} = -I_3$.

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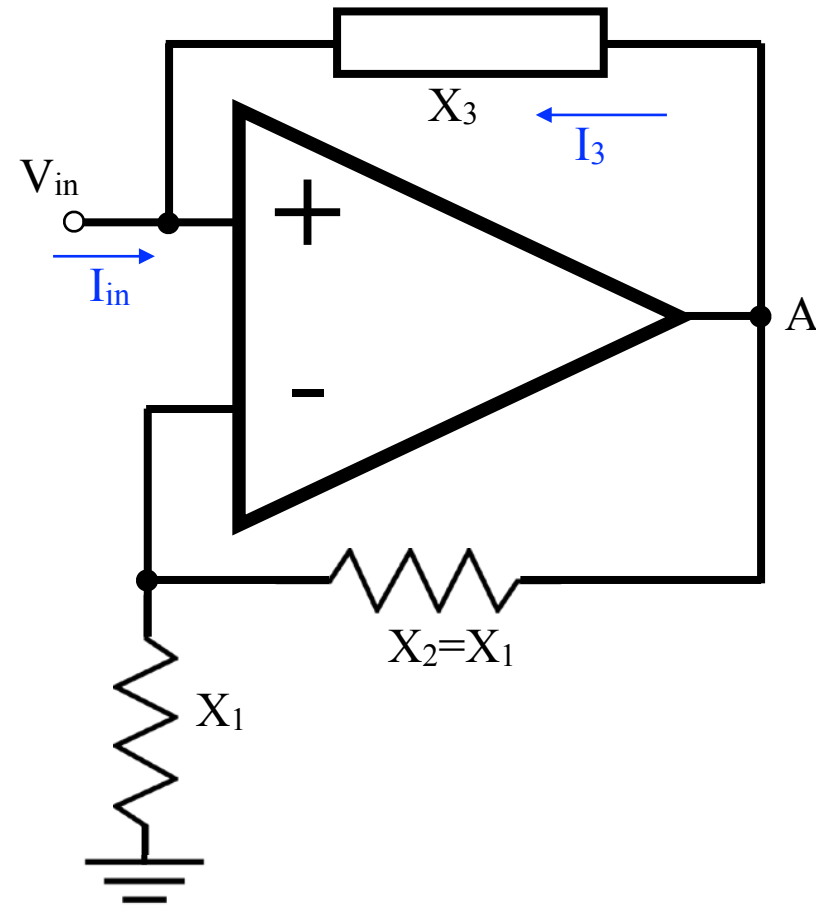
$$X_{in} = \Delta V_{in} / \Delta I_{in} = \Delta V_{in} / (-\Delta V_{in} X_2 / (X_1 X_3))$$

$$X_{in} = - X_1 X_3 / X_2$$

Negative impedance converter

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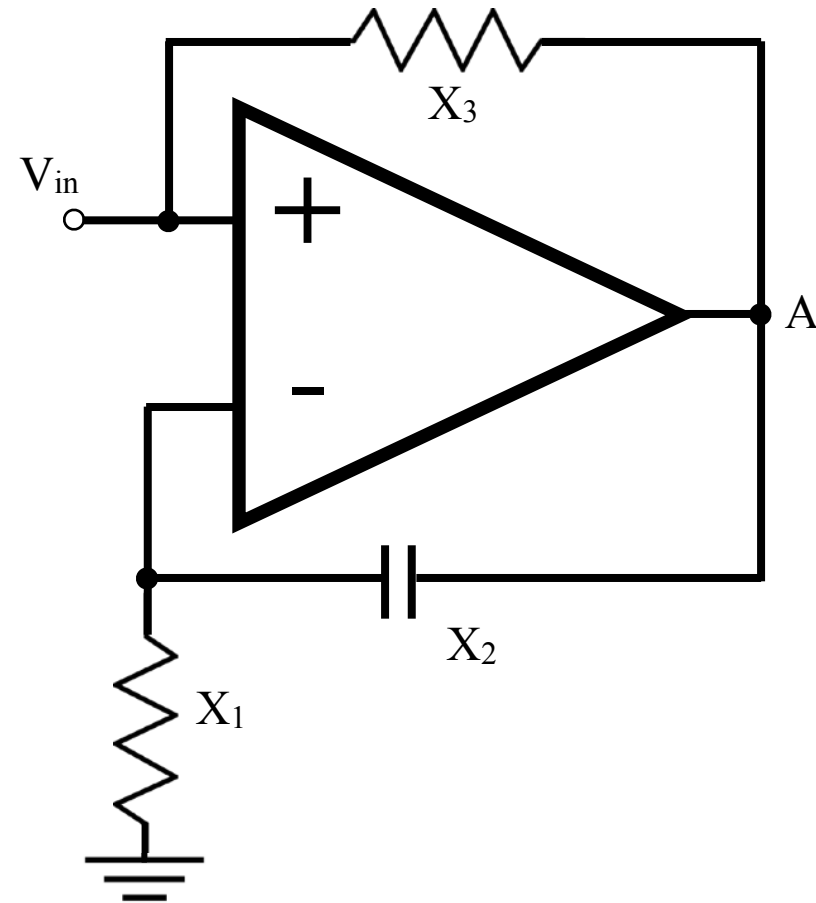
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$$X_{in} = - X_1 X_3 / X_2$$

$$X_{in} = - R R / (-j/\omega C) = R^2 \omega C / j$$

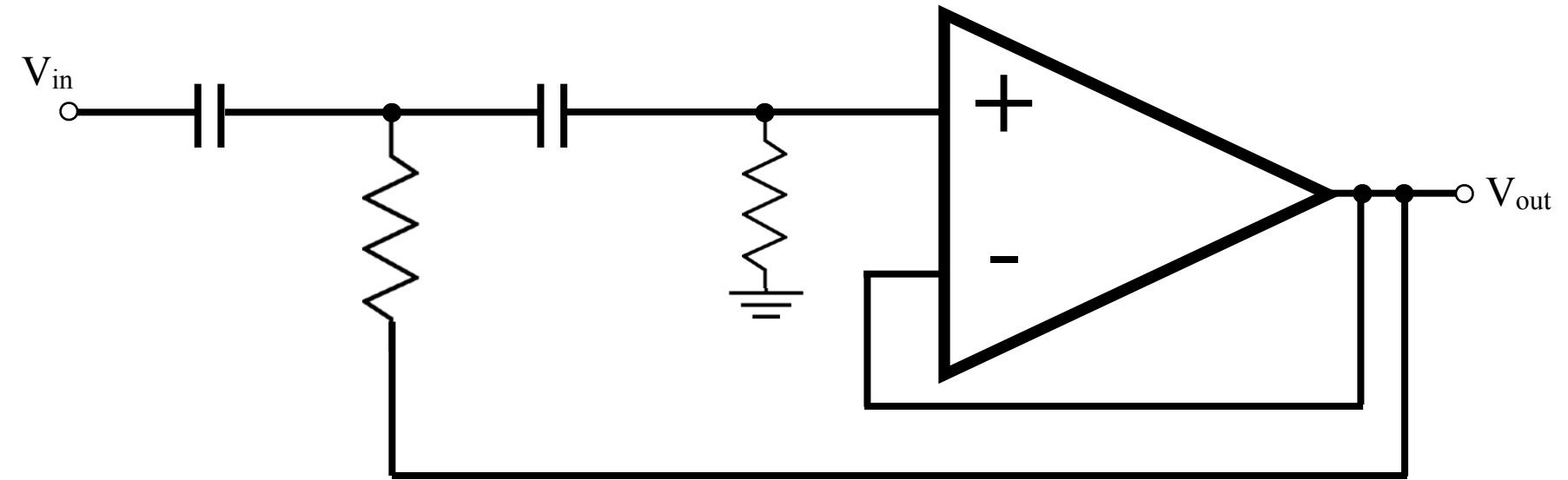
$$X_{in} = -j\omega C R^2$$

Combine this with previous, plug it in using the fact that *ground is just a reference*.

$X = j\omega C R^2$ which is like an $L = C R^2$.

Sallen-Key filter configuration

Can combine R's, C's, and op-amps in general configuration.

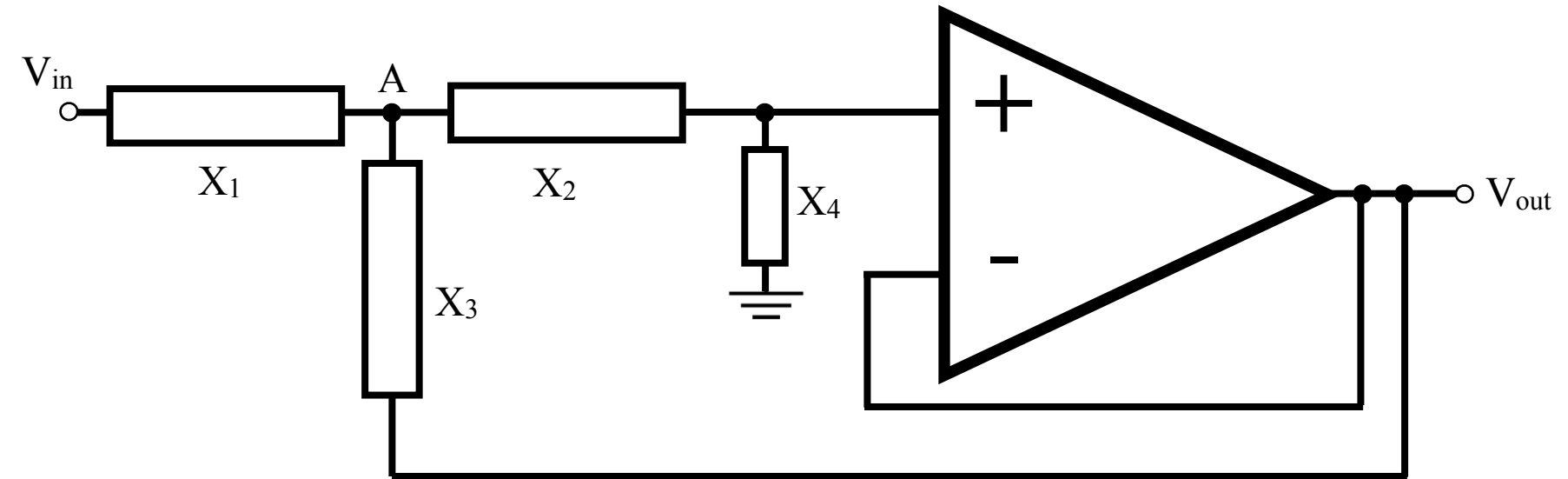


Analyzing the V_{out} vs V_{in} behavior can be done with the golden rules.

But let's do that more generally.

Voltage-controlled voltage source

Can combine R's, C's, and op-amps in general configuration.



Analyzing the V_{out} vs V_{in} behavior can be done with the golden rules and Ohm's law.

$$V_+ = ?$$

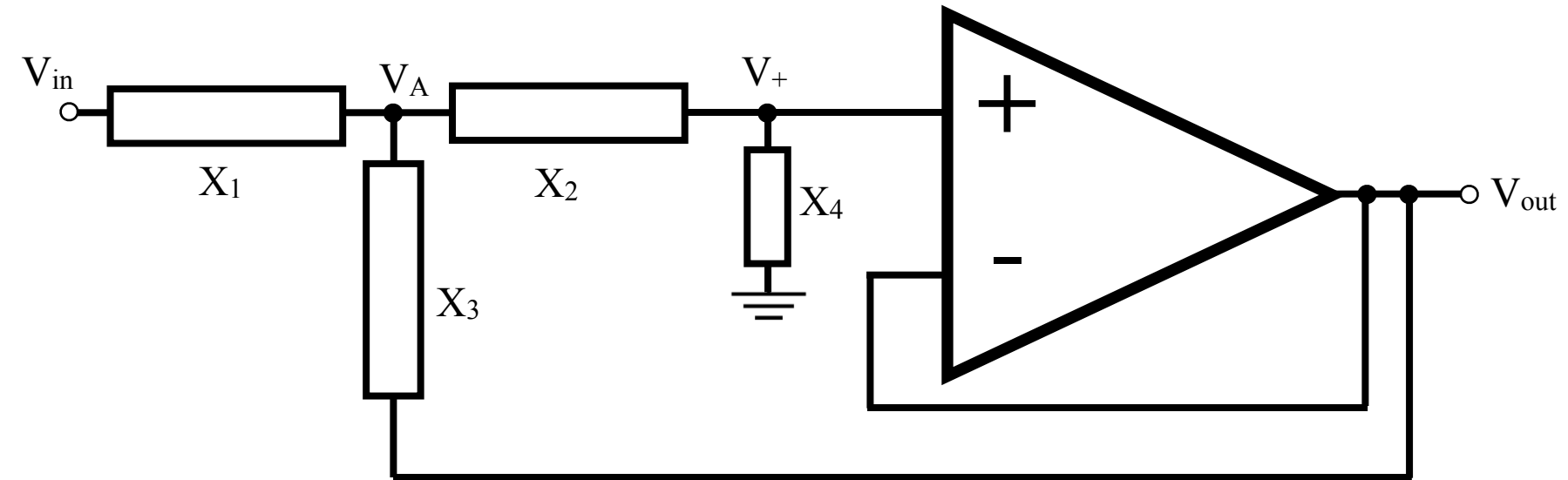
$$V_A = ?$$

$$V_A - V_+ = ?$$

$$V_+ = ?$$

Voltage-controlled voltage source

Can combine R's, C's, and op-amps in general configuration.



Analyzing the V_{out} vs V_{in} behavior can be done with the golden rules and Ohm's law.

$$V_+ = V_{out}$$

$$V_A = V_{in} - I_1 X_1$$

$$V_A - V_+ = I_2 X_2$$

$$V_+ = I_4 X_4 = I_2 X_4$$

$$V_A - V_{out} = I_3 X_3$$

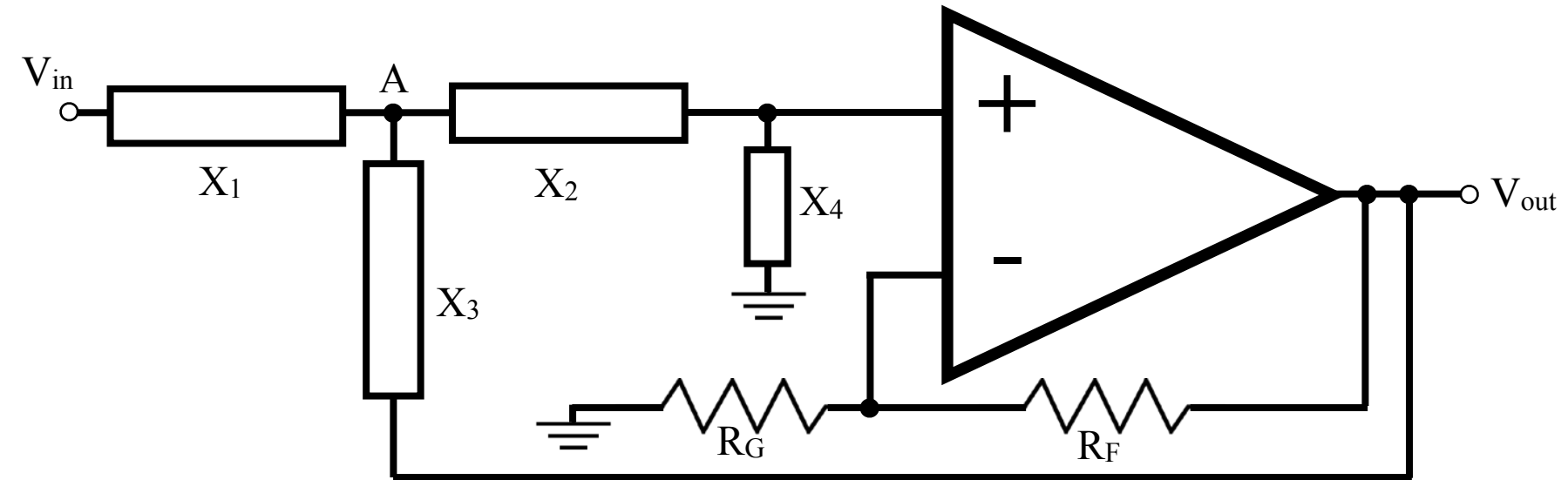
$$V_+ = V_A X_4 / (X_2 + X_4)$$

$$\frac{V_{out}}{V_{in}} = \frac{X_3 X_4}{X_1 X_2 + X_3 (X_1 + X_2) + X_3 X_4}$$

You can choose any mix of R and C (or even L with NIC) to get whatever relationship you want.

Voltage-controlled voltage source

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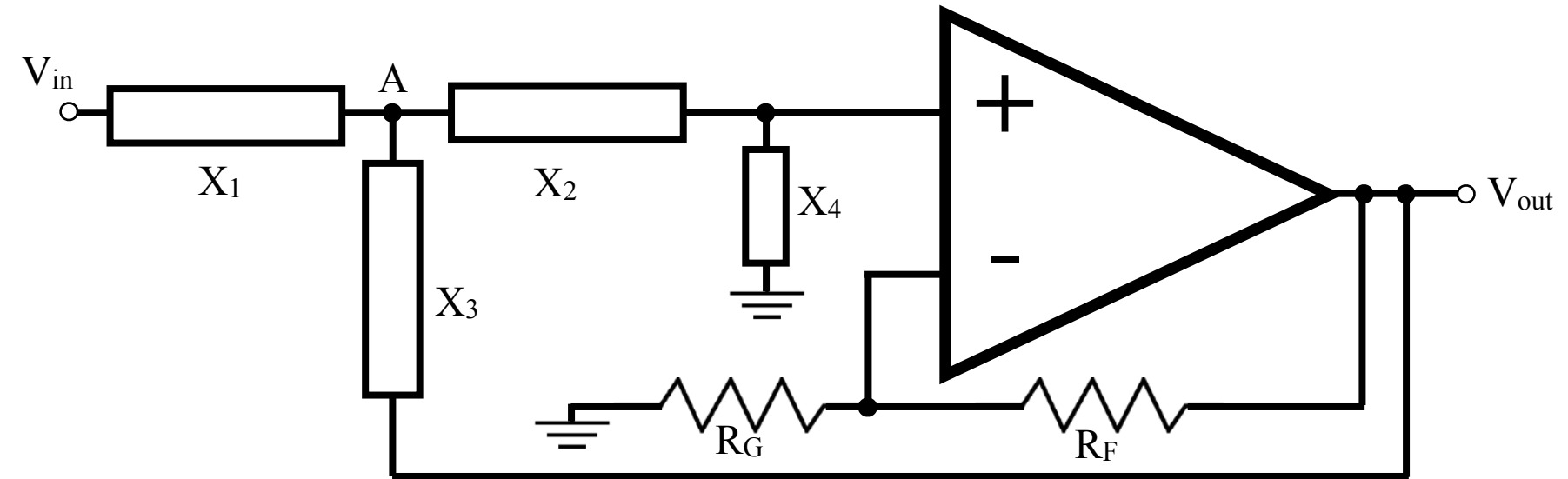
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You can choose any mix of R and C (or even L with NIC) to get whatever relationship you want.

We could also add amplification.

Voltage-controlled voltage source

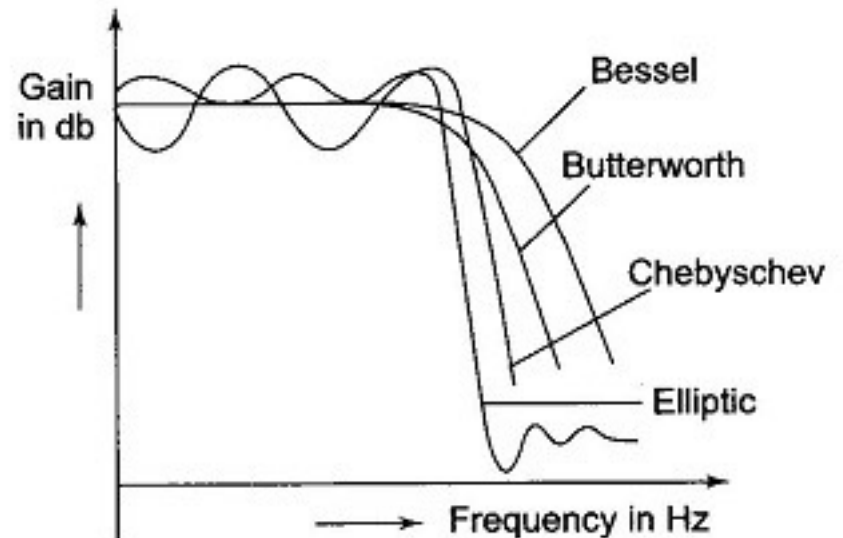
Can combine R's, C's, and op-amps in general configuration.



Can vary $X_1 - X_4$ and gain to get a variety of different response types.

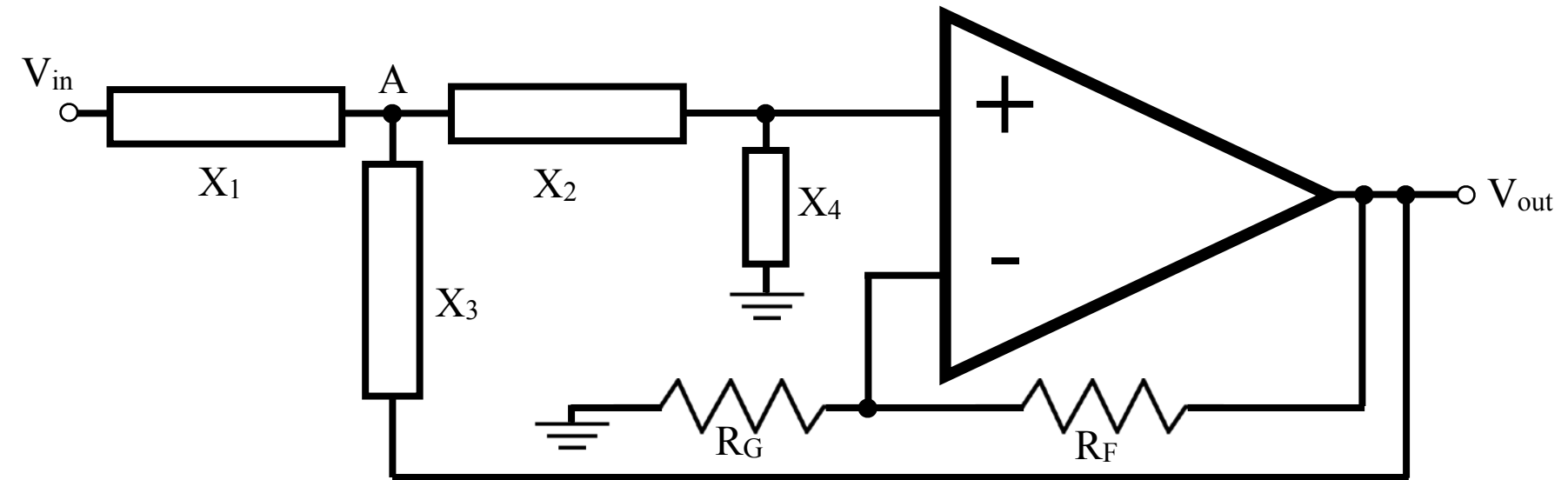
You will not be likely to build your own active filter.

Can buy them to match specs.



Voltage-controlled voltage source

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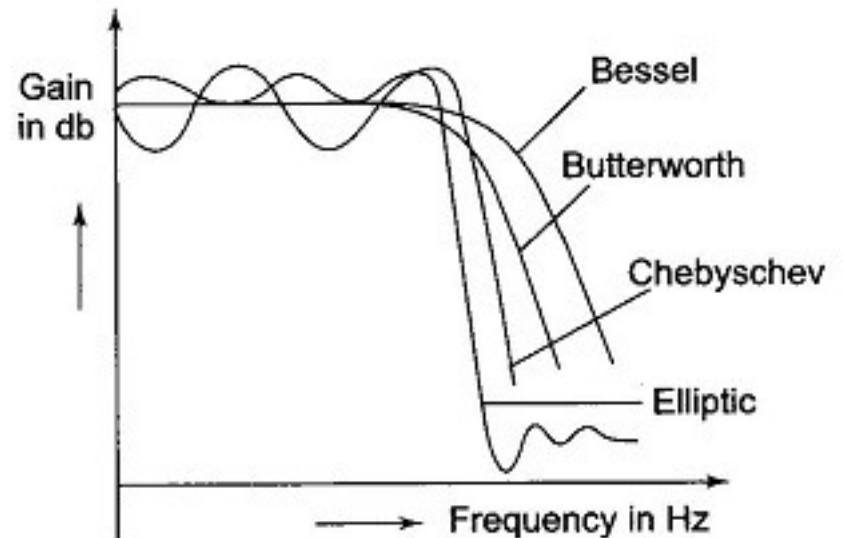


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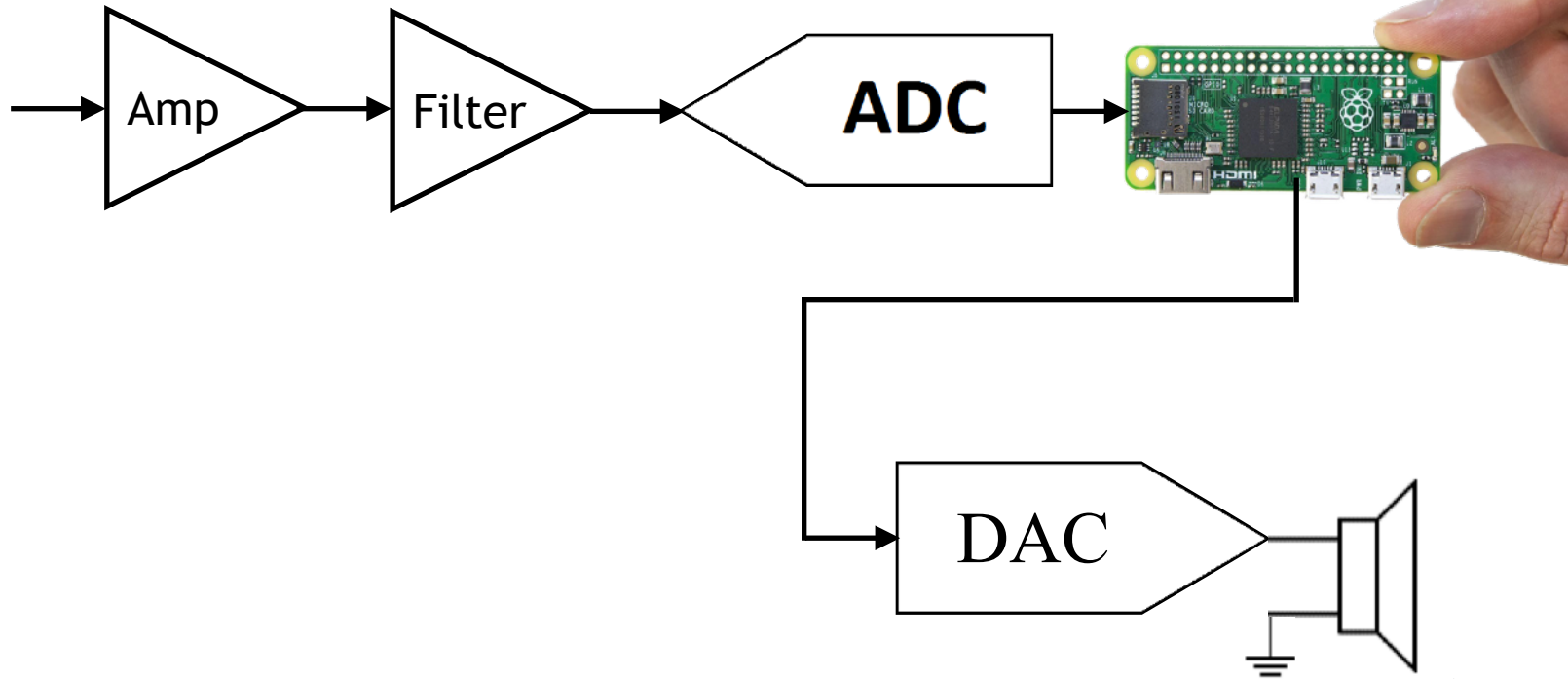
Can buy them to match specs.

But they can cost \$5 - \$10.



Software or firmware digital signal processing

It is getting more common to simply sample, digitally process, and then re-drive.

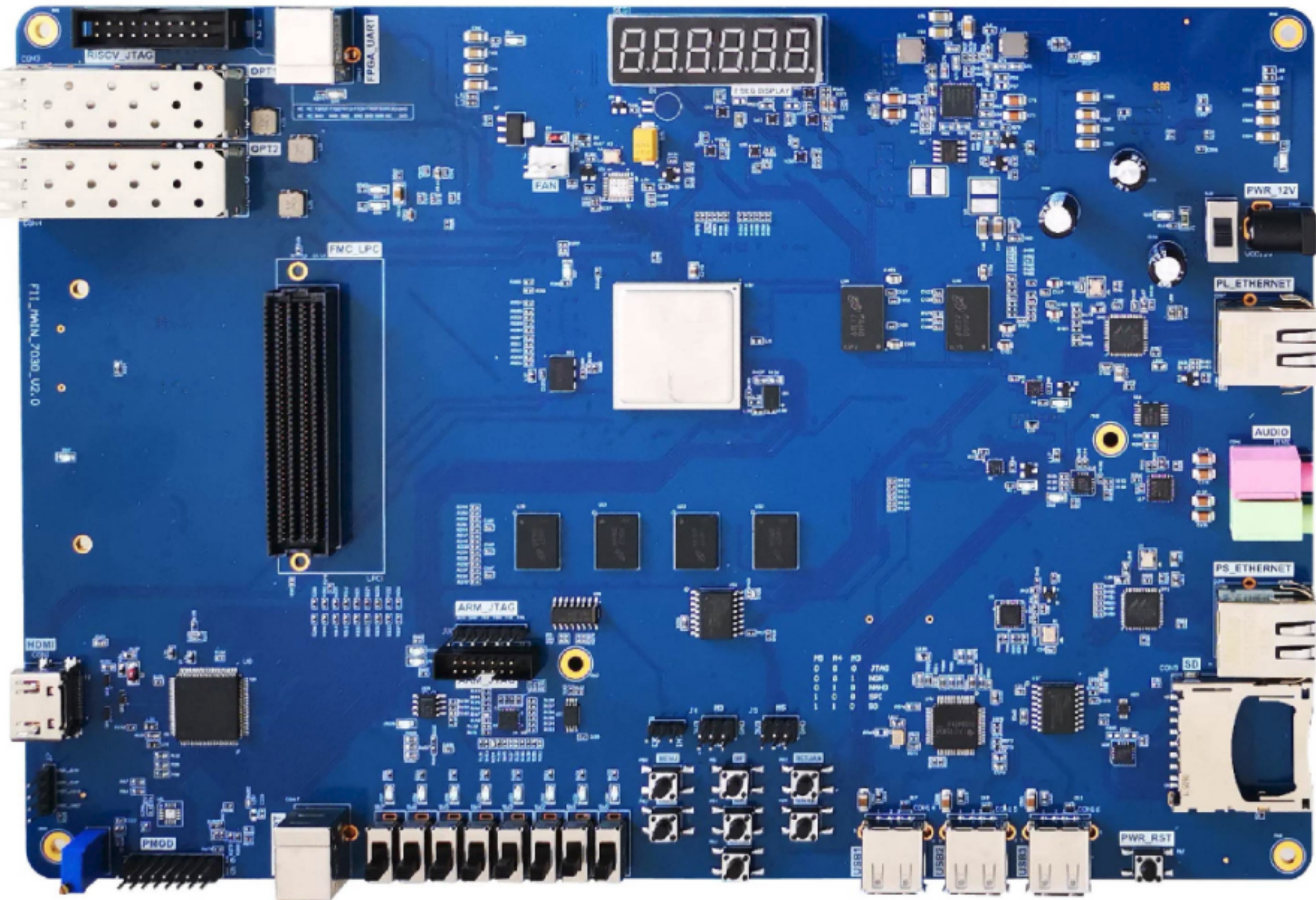


Software or firmware digital signal processing

It is getting more common to simply sample, digitally process, and then re-drive.

For processing a lot of parallel data, can use a Field Programmable Gate Array (FPGA) which can process hundreds of inputs at O(GHz) in parallel.



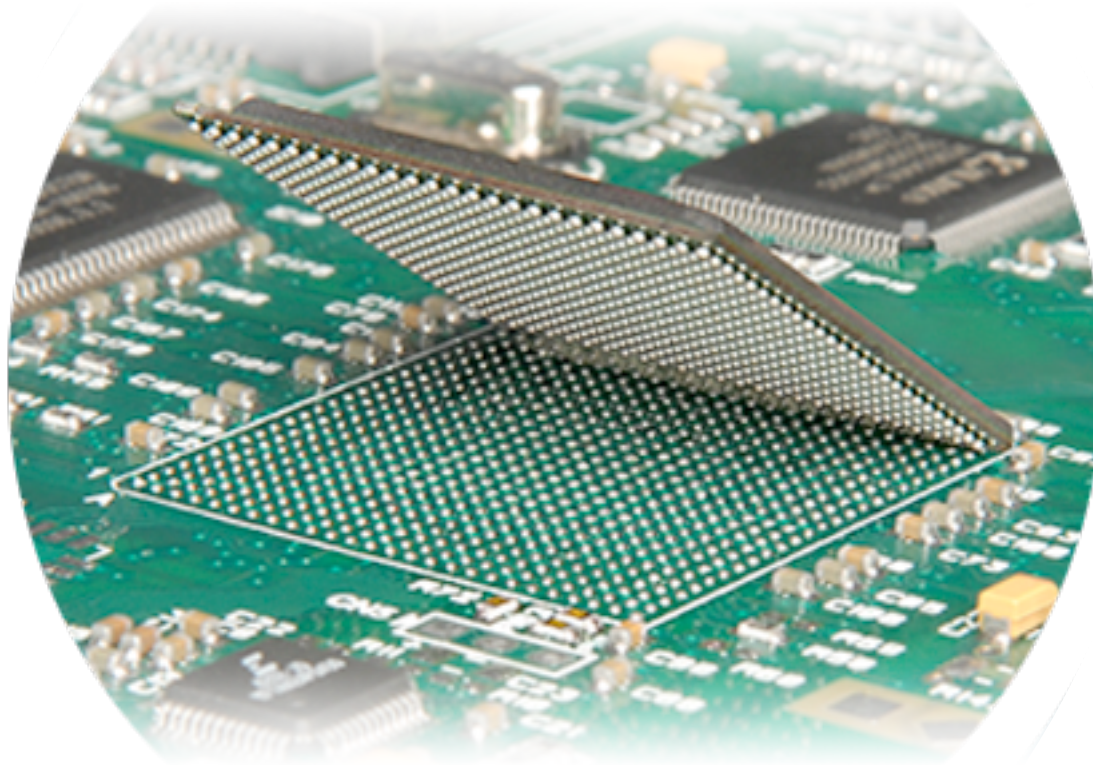


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Many inputs, so have a 2D grid of connections with sub-mm spacing.

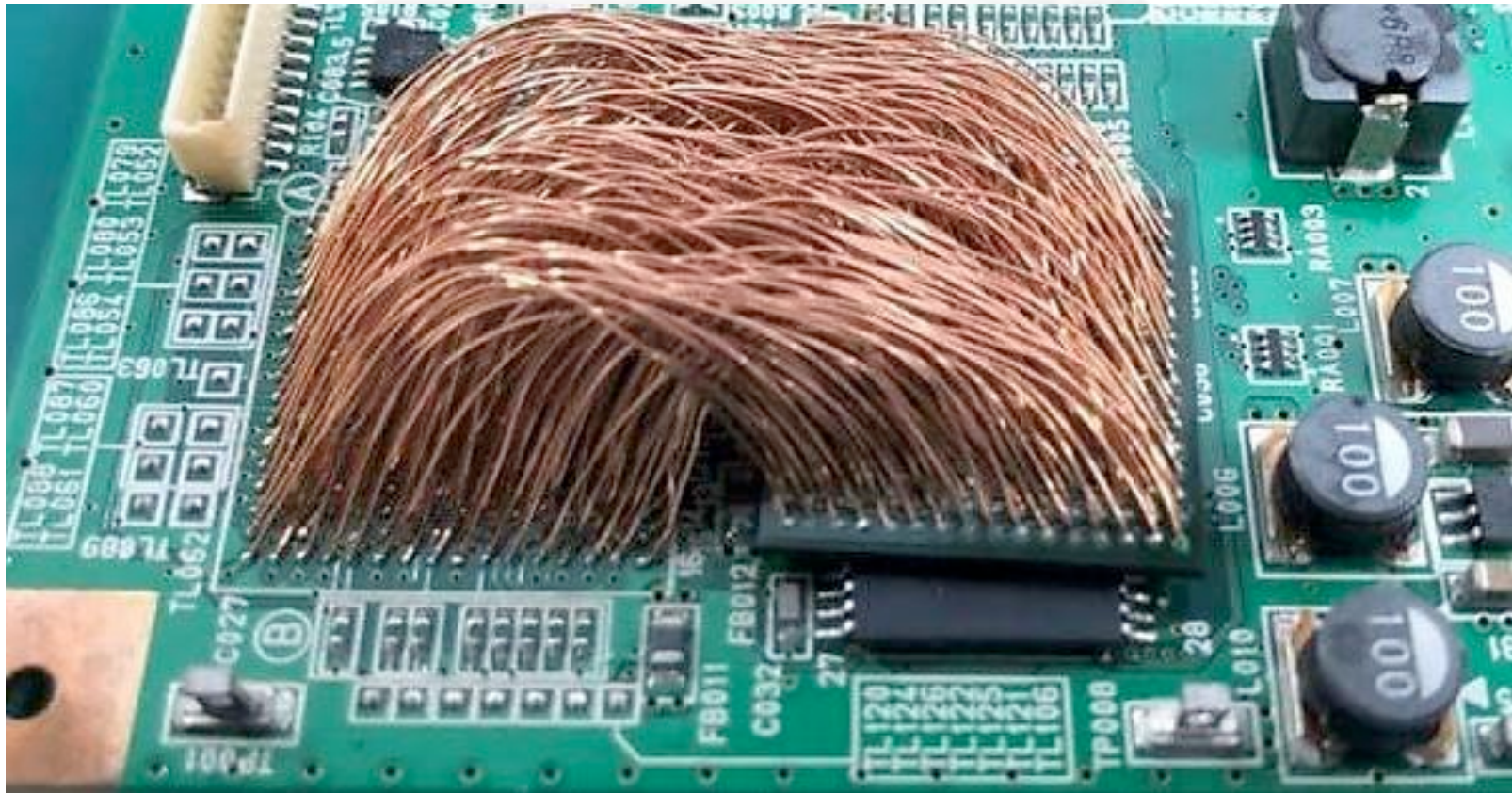


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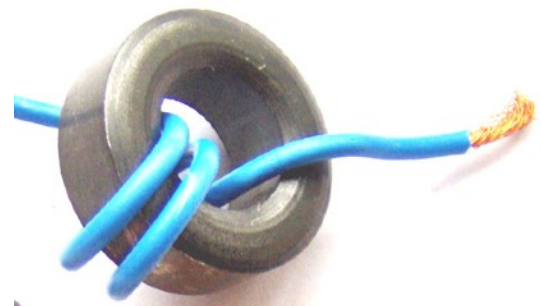
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For processing a lot of parallel data, can use a Field Programmable Gate Array (FPGA) which can process hundreds of inputs at O(GHz) in parallel.

Many inputs, so have a 2D grid of connections with sub-mm spacing.

But “better is the enemy of good.”

You can often just think about it and “add a cap” or an inductor to make it work.



As a hint toward next time: What does this circuit do?

